Polish Academy of Sciences

Multiple Criteria Decision Aiding by Constructive Preference Learning

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Plan

- Introduction – where is the challenge?
- Why Multiple Criteria Decision Aiding?
- Evolution of MCDA influenced by the Artificial Intelligence paradigm
- Robust ordinal regression for value function preference model
  - Extreme ranking analysis
  - Stochastic ordinal regression
  - Robust Ordinal Regression for hierarchy of criteria
- Robust ordinal regression for outranking relation preference model
- Robust ordinal regression for decision rule preference model
- Examples of applications
- Summary and conclusions

Introduction – where is the challenge?

Decision problem

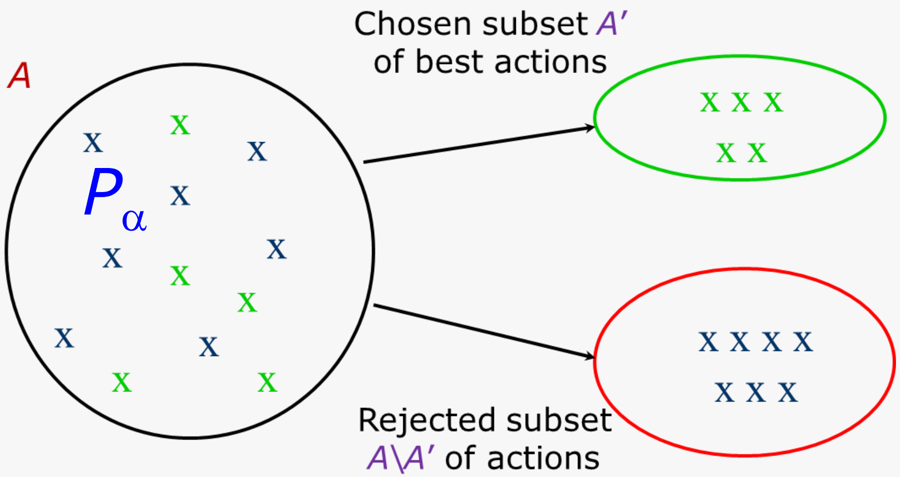
- There is an objective or objectives to be attained
- There are many alternative ways for attaining the objective(s) – they consititute a set of actions A (alternatives, solutions, objects, acts, …)
- Questions with respect to set A:

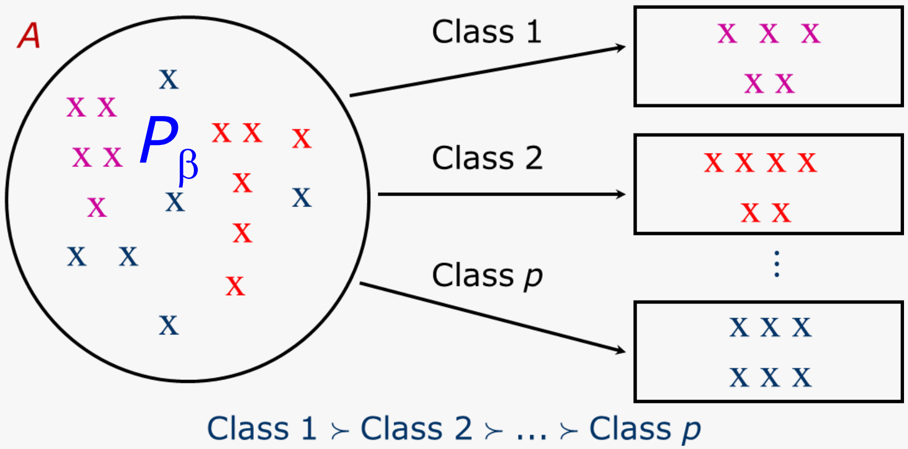
  P_α: *How to choose the best action* ?

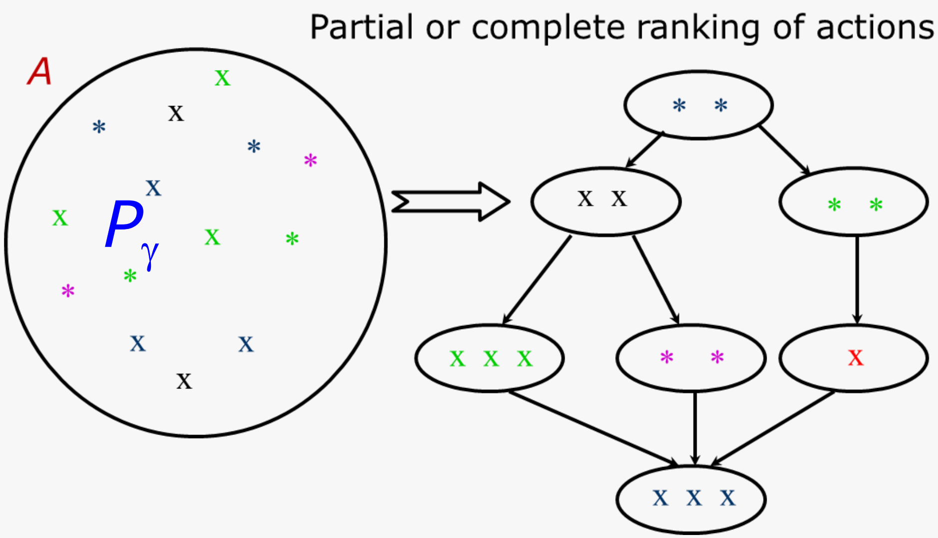
  P_β : *How to classify actions into pre-defined decision classes* ?

  P_γ : *How to order actions from the best to the worst* ?

Decision problem







Coping with multiple dimensions in Decision Aiding

- Decision problems P_α, P_β , P_γ involve vector evaluations of actions coming from:
  - multiple decision makers (voters, group decision)
  - multiple evaluation criteria (multiple objectives)
  - multiple possible states of the world that imply multiple consequences of the actions (probabilities of outcomes)

Multi-dimensional decision problems

	Social Choice (Group Decision)	Multiple Criteria Decision Aiding	Decision under Risk and Uncertainty
Element of set *A*	Candidate	Action	Act
Dimension of evaluation space	Voter	Criterion	Probability of an outcome
Objective information about comparison of elements from *A*	Dominance relation	Dominance relation	Stochastic dominance relation

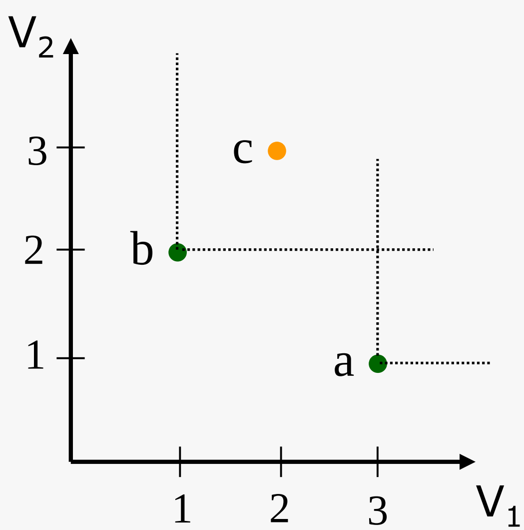
- The only objective information one can draw from the statement of a multi-dimensional decision problem is the **dominance relation**

SC&GD

Cand.	Voters: V_1	V_2
a	3	1
b	1	2
c	2	3

$V_1 : b \succ c \succ a$

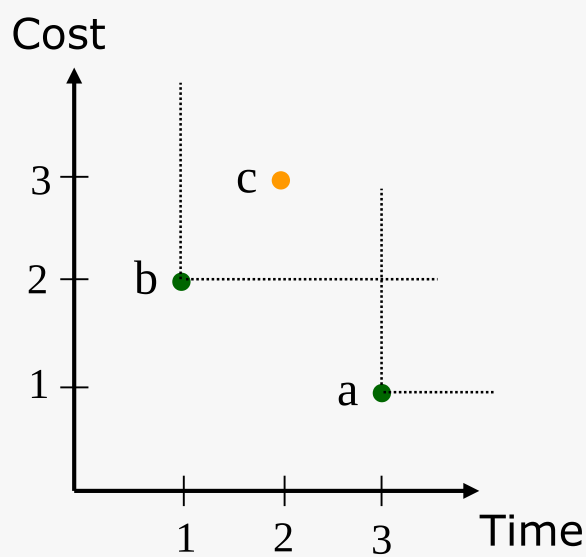
$V_2 : a \succ b \succ c$



MCDA

Action	Criteria: Time	Cost
a	3	1
b	1	2
c	2	3

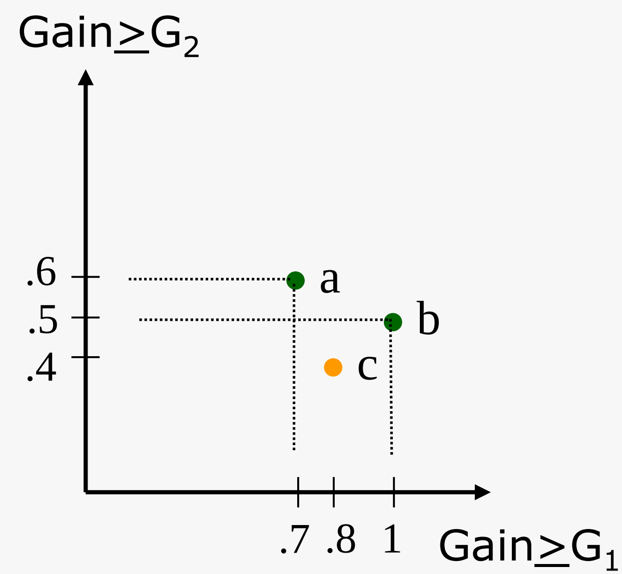
- non-dominated
- dominated



DRU

Act	Probability of gain: Gain$\geq G_1$	Gain$\geq G_2$
a	0.7	0.6
b	1.0	0.5
c	0.8	0.4

$G_1 < G_2$



Enriching dominance relation – preference modeling/learning

- Dominance relation is too poor – it leaves many actions non-comparable

1

- One can „enrich" the dominance relation, using preference information elicited from the DM

2

- Preference information is an input to learn/build a preference model that aggregates the vector evaluations of actions
- The preference model induces a preference relation in set *A*, richer than the dominance relation (the elements of *A* become more comparable)

3

- A proper exploitation of the preference relation in *A* leads to a recommendation in terms of choice, classification or ranking
- In this talk, we will consider multiple criteria decision aiding

Aggregation of multiple criteria evaluations – preference models

- Three families of **preference modeling (aggregation) methods**:
  - Multiple Attribute Utility Theory (MAUT) using a value function,

    e.g., $U(a)=\sum_{i=1}^{n} w_i g_i(a)$, $U(a)=\sum_{i=1}^{n} u_i[g_i(a)]$, Choquet/Sugeno integral

  - Outranking methods using an outranking relation $S=\{\sim\cup\succ^{w}\cup\succ^{s}\}$

    $a\ S\ b$ = „a is at least as good as b"

  - Decision rule approach using a set of decision rules

e.g., „If $g_i(a)\succeq r_i$ & $g_j(a)\succeq r_j$ & ... $g_h(a)\succeq r_h$, then $a \rightarrow$ *Class t* or higher"

„If $g_i(a)\succeq_i^{\geq h(i)} g_i(b)$ & $g_j(a)\succeq_j^{\geq h(j)} g_j(b)$ & ... $g_p(a)\succeq_p^{\geq h(p)} g_p(b)$, *then* aSb"

  - Decision rule model is the most general of all three

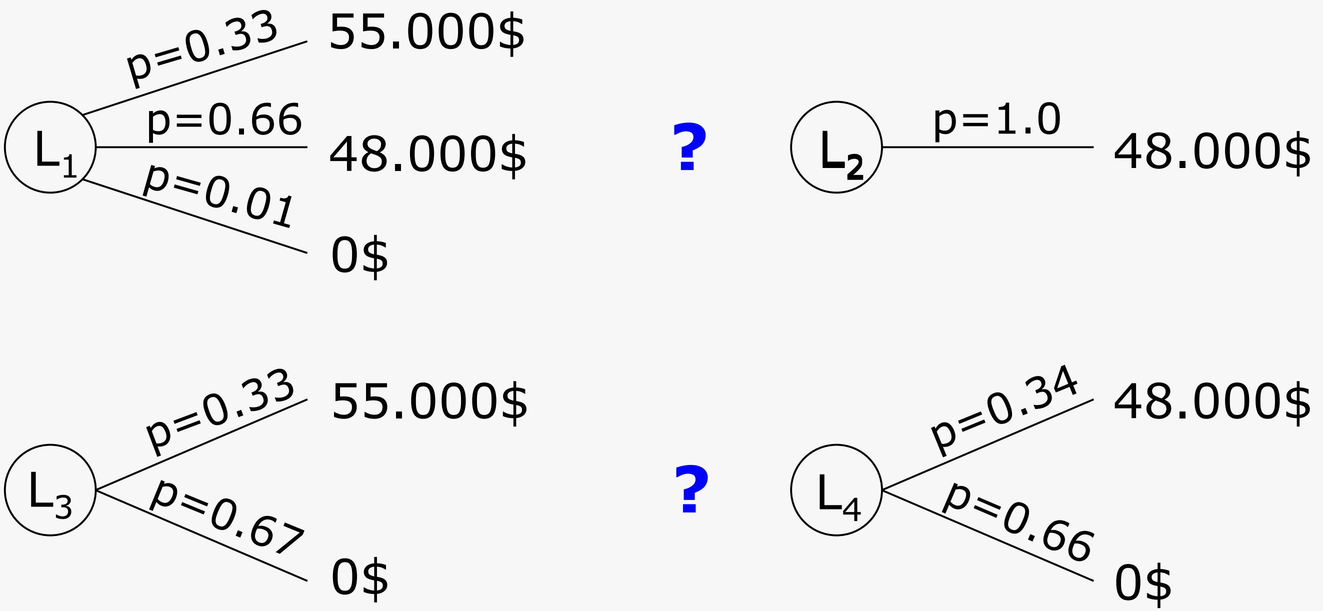
R. Słowiński, S. Greco, B. Matarazzo: Axiomatization of utility, outranking and decision-rule preference models for multiple-criteria classification problems under partial inconsistency with the dominance principle, *Control & Cybernetics*, 31 (2002) no.4, 1005-1035

Multiple-criteria approach over mono-criterion approach

- Operations Research was originally focused on mono-criterion optimization – mathematical programming, MAUT (utility function)
- A decision maker (DM) seldom has a single clear criterion in mind. Usually, there is no common unit for all scales of criteria, which are rather heterogeneous, so it may be very difficult to define a priori a unique criterion able to take into account all relevant points of view
- By making a family of criteria explicit, the multiple-criteria approach preserves the original concrete meaning of the corresponding evaluations for each actor, without resorting to any fabricated conversion (*the nightmare of composite indicators*)

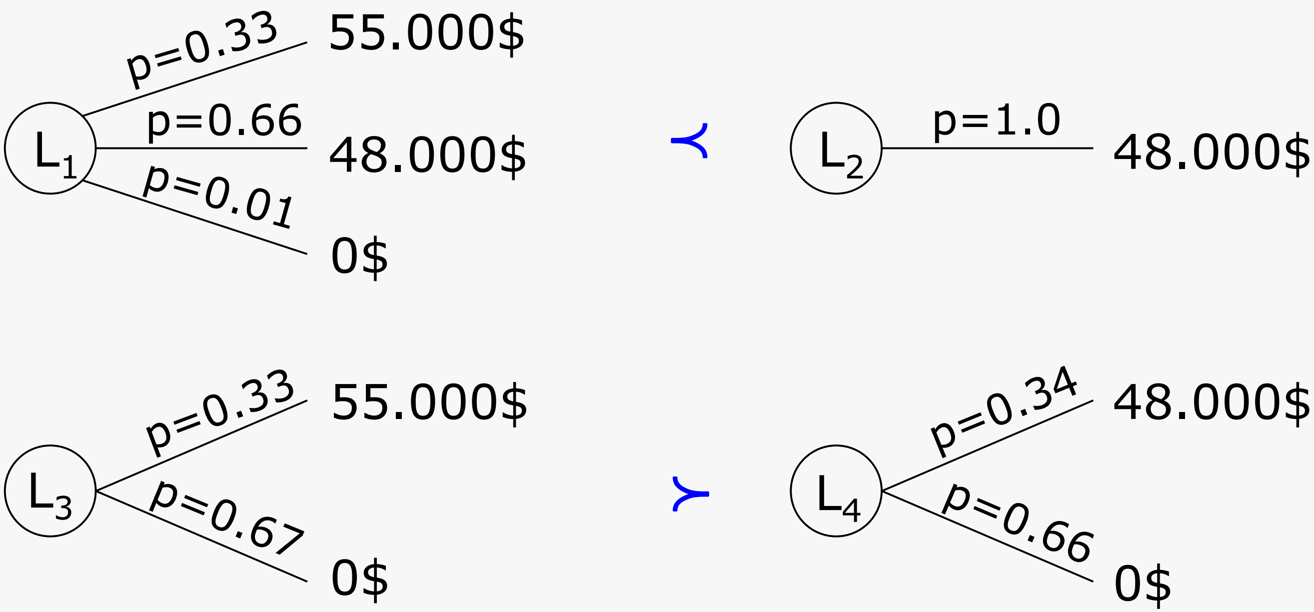
Von Neumann-Morgenstern utility function

- Experiments show systematic violation of expected utility hypotheses



Von Neumann-Morgenstern utility function – certainty effect

Typical response



- expected utility function is linear in the probabilities

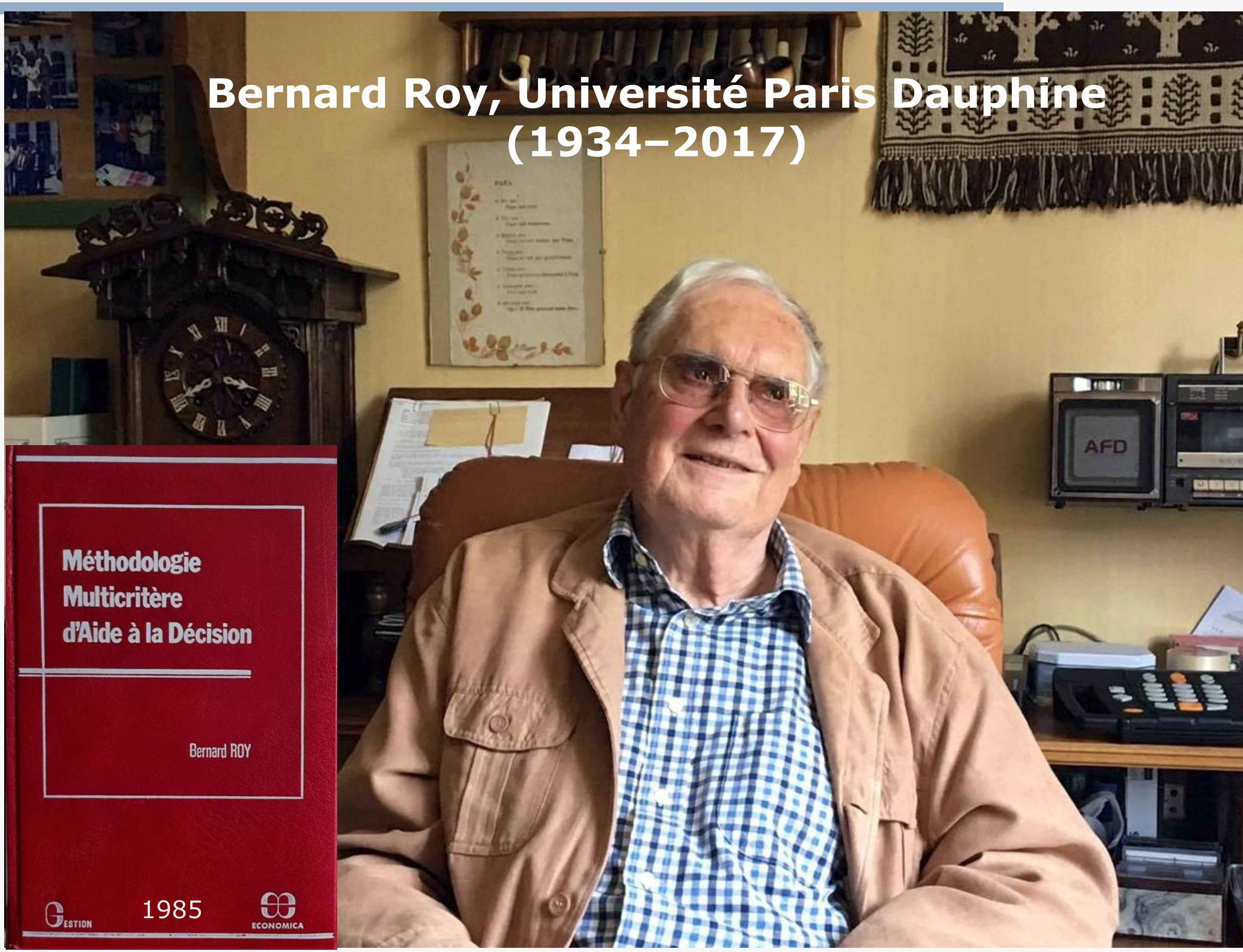
$$u_i(x_i^3) = p_i\, u_i(x_i^1) + (1-p_i)\, u_i(x_i^2)$$

$$U(48) > 0.33 \times U(55) + 0.66 \times U(48) + 0.01 \times 0 \quad \Rightarrow 0.34 \times U(48) > 0.33 \times U(55)$$

$$0.33 \times U(55) + 0.67 \times 0 > 0.34 \times U(48) + 0.66 \times 0 \Rightarrow 0.34 \times U(48) < 0.33 \times U(55)$$

- Kahneman & Tversky: people tend to overvalue a sure thing

Multiple Attribute Utility Theory vs. Multiple Criteria Decision Aiding



Main sources of imperfect knowledge and ill determination (Roy 1985)

1. Roy’s staring hypothesis was that realistic decision aidiong takes place in the context of imperfect knowledge and ill determination

2. The decision aiding process is carried out in a real life context that may not correspond exactly to the model on which the decision aiding is based (*the map is not the territory*)

3. The system of values used for evaluating the feasibility and relative interest of diverse potential actions is usually fuzzy, incomplete and influenceable

4. Hesitation of the DM, instability of their preferences, absence of some hardly expressible criteria in the considered family make that people in their judgments violate dominance

5. Preference information is inconsistent, vague and ambiguous

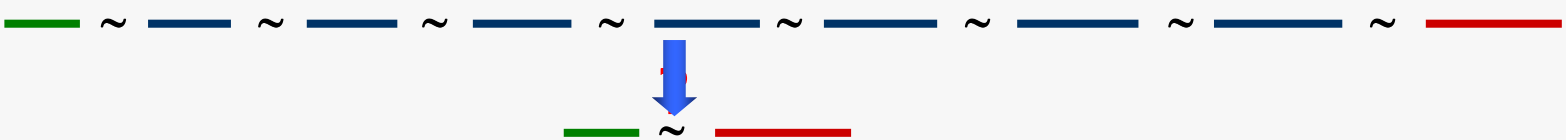
Weak points of the aggregation by utility (value) function (MAUT)

- Utility function distinguishes only 2 possible relations between actions:

  preference relation: $a \succ b \Leftrightarrow U(a) > U(b)$

  indifference relation: $a \sim b \Leftrightarrow U(a) = U(b)$

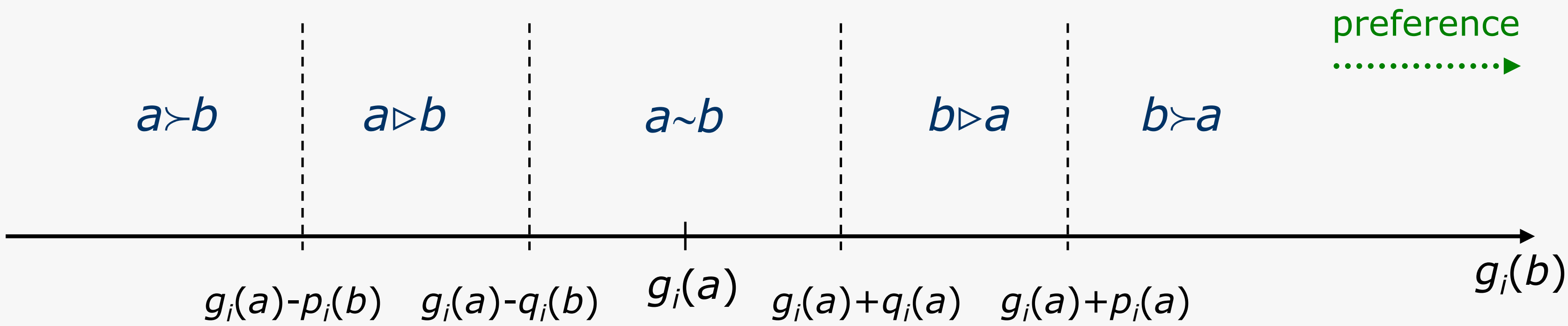
- $\succ$ is asymmetric (antisymmetric and irreflexive) and transitive
- $\sim$ is symmetric, reflexive and transitive
- Transitivity of indifference is troublesome, e.g.



- In consequence, a non-zero indifference threshold q_i is necessary
- An immediate transition from indifference to preference is unrealistic, so a preference threshold $p_i \geq q_i$ and a weak preference relation $\triangleright$ are desirable
- Another realistic situation which is not modelled by U is incomparability, so a good model should include also an incomparability relation „?”

Four basic preference relations and an outranking relation S

- Four basic preference relations are: $\{\sim, \rhd, \succ, ?\}$



- Outranking relation S groups three basic preference relations:

$$S = \{\sim, \rhd, \succ\} \text{ – reflexive and non-transitive}$$

aSb means: „action a is at least as good as action b"

- For each couple $a,b \in A$:

$$aSb \wedge \text{non } bSa \iff a \rhd b \vee a \succ b$$

$$aSb \wedge bSa \iff a \sim b$$

$$\boxed{\text{non } aSb \wedge \text{non } bSa \iff a?b}$$

The evolution of MCDA towards AI

- Aggregation of vector evaluations, i.e., preference modeling:
  - till early 80’s: „model-centric”
    (model first, then preference info in terms of model parameters)
  - since 80’s: more and more „human-centric”
    (PC allowed human-computer interaction – „trial-an-error”)
  - in XXI century: „knowledge driven”
    (more data about human choices;
    holistic preference information first, then model building;
    explanation of past decisions, and prediction of future decisions;
    AI – model and human learn in the loop of interaction)

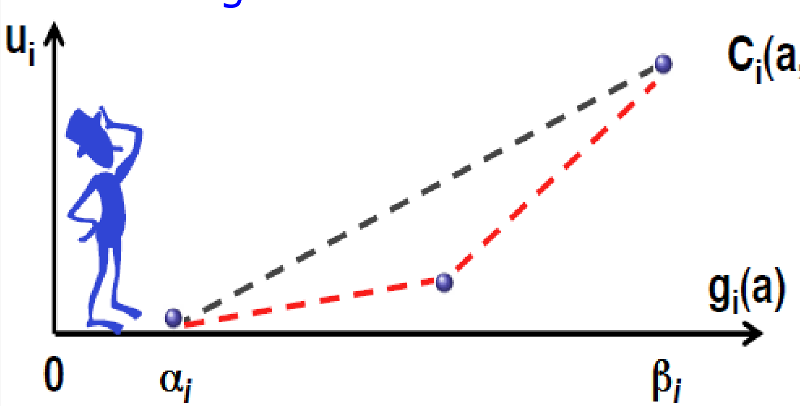
Elicitation of preference information by the Decision Maker (DM)

- Direct or indirect ?
- Direct elicitation of numerical values of model parameters by DMs demands much of their cognitive effort

P.C.Fishburn (1967): Methods of Estimating Additive Utilities. *Management Science*, 13(7), 435-453 (listed and classified twenty-four methods of estimating additive utilities)

Value function model

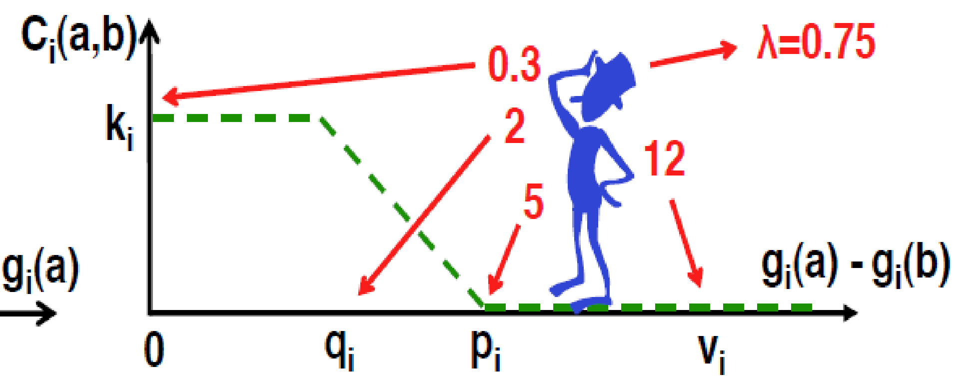
substitution rates or shapes of marginal value functions



$$U(a)=\sum_{i=1}^{n} w_i g_i(a) \ \text{ or } \ \sum_{i=1}^{n} u_i[g_i(a)]$$

Outranking model

weights & discrimination thresholds



$$aSb \Leftrightarrow C(a,b)=\sum_{i=1}^{n} C_i(a,b) \ge \lambda$$

and $g_i(b)-g_i(a) \le v_i$ for all i

Elicitation of preference information by the Decision Maker (DM)

- Indirect elicitation: through holistic judgments, i.e., decision examples
- Decision aiding based on decision examples is gaining importance because:
  - Decision examples are relatively „easy" preference information
  - Decisions can also be observed without active participation of DMs
  - Psychologists confirm that DMs are more confident exercising their decisions than explaining them (J.G.March 1978; P.Slovic 1977)
- Related paradigms:
  - Revealed preference theory in economics (P.Samuelson 1938), is a method of analyzing choices made by individuals: preferences of consumers can be revealed by their purchasing habits
  - Learning from examples in AI/ML (knowledge discovery)
- Conclusion: indirect elicitation of preferences is more user-friendly

Indirect elicitation of preference information by the DM

[TIME=24, COST=56, RISK=75]

$\succ$

[TIME=28, COST=67, RISK=25]

Pairwise preferences between actions

$\succ$

$\succ$

characterized by cardinal and/or ordinal features (criteria)

[MATH=18, PHYS=16, LIT=15] $\Rightarrow$ Class „MEDIUM”
[MATH=17, PHYS=16, LIT=18] $\Rightarrow$ Class „GOOD”

Classification examples

A is preferred to **Z** more than **C** is preferred to **K**

Intensity of preference

Action **F** should be among **5%** of the best ones

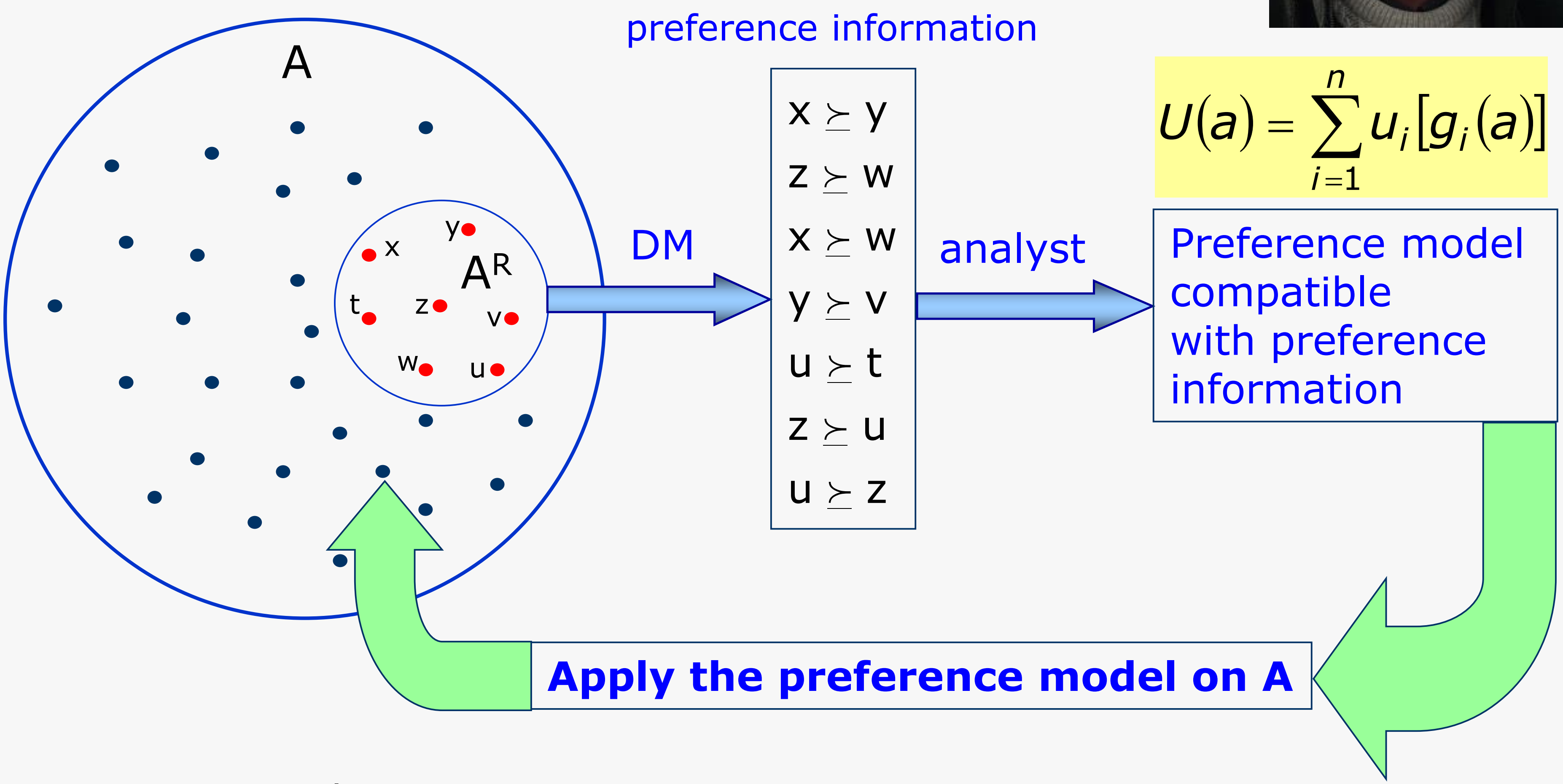
Rank related

Ordinal regression paradigm

Eric Jacquet-Lagrèze (1947–2017)

Ordinal regression paradigm (UTA method)

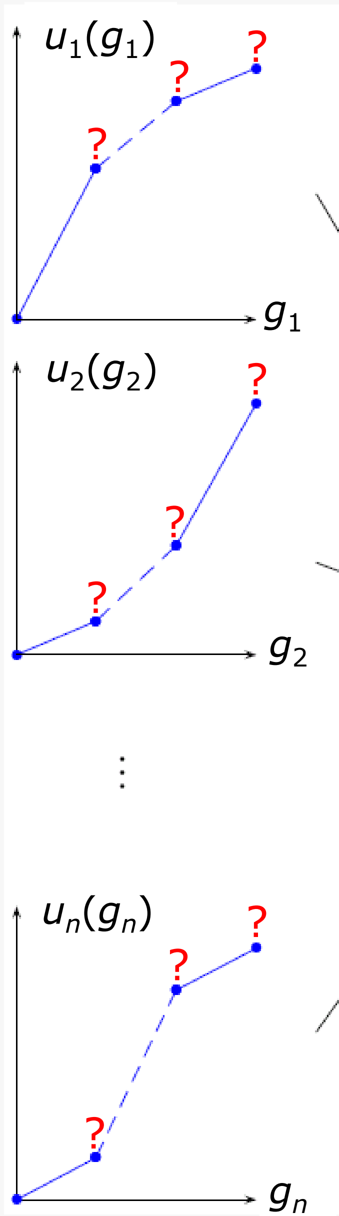
- Ordinal regression paradigm emphasizes the discovery of intentions expressed through decision examples



E. Jacquet-Lagrèze, J. Siskos: Assessing a set of additive utility functions for multicriteria decision-making, the UTA method. *Europ. J. Operational Research*, 10 (1982) 151-164

UTA additive preference model

Marginal value functions $u_i(g_i)$



The scale of u_i is a conjoint interval scale whatever the scale of g_i

? can be found by LP

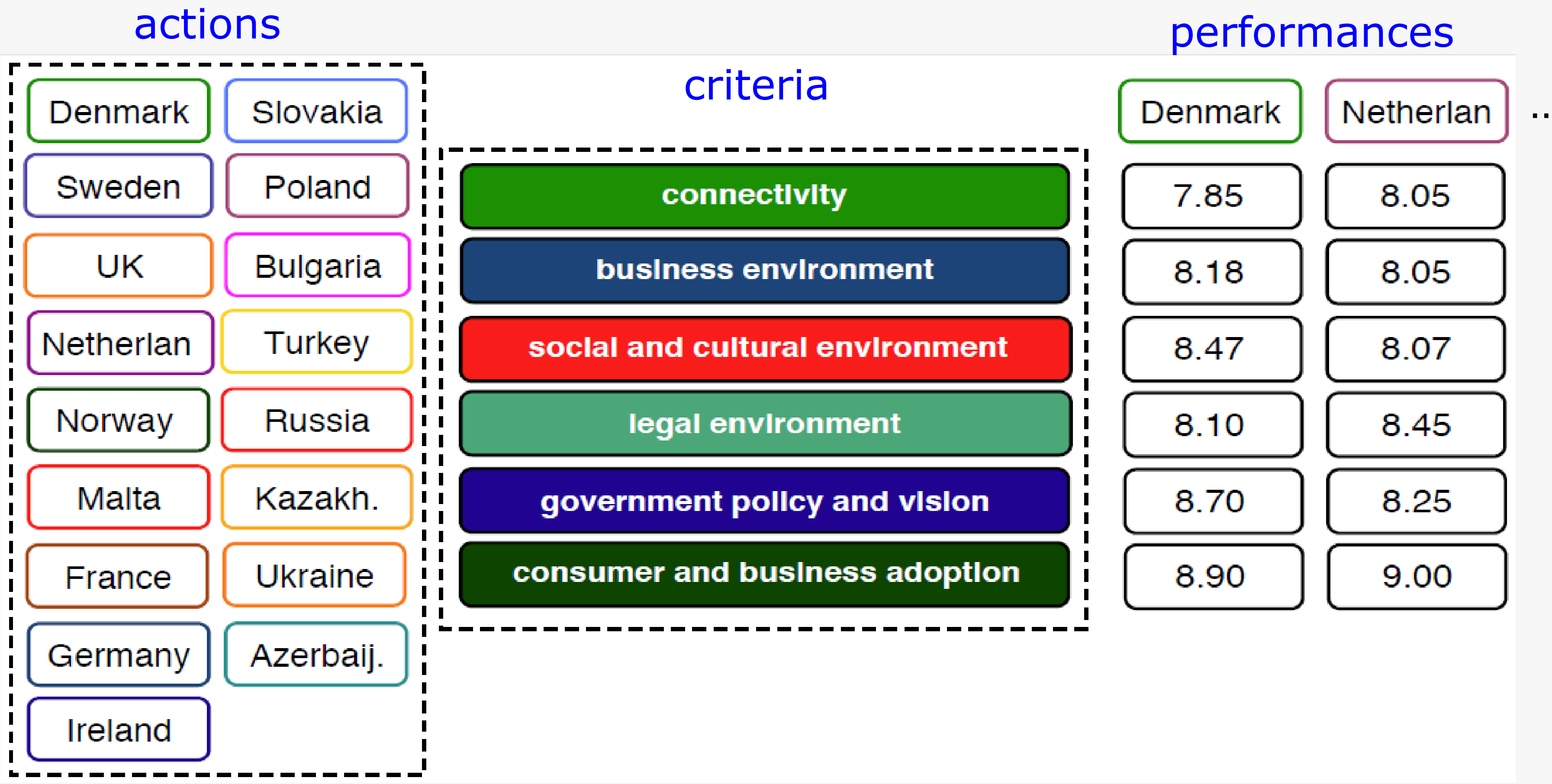
$$U(x) = \sum_{i=1}^{n} u_i[g_i(x)]: \text{ the value of action } x$$

having evaluations $g_i(x), \; i = 1,\dots,n$

Criteria are supposed to be independent with respect to preferences

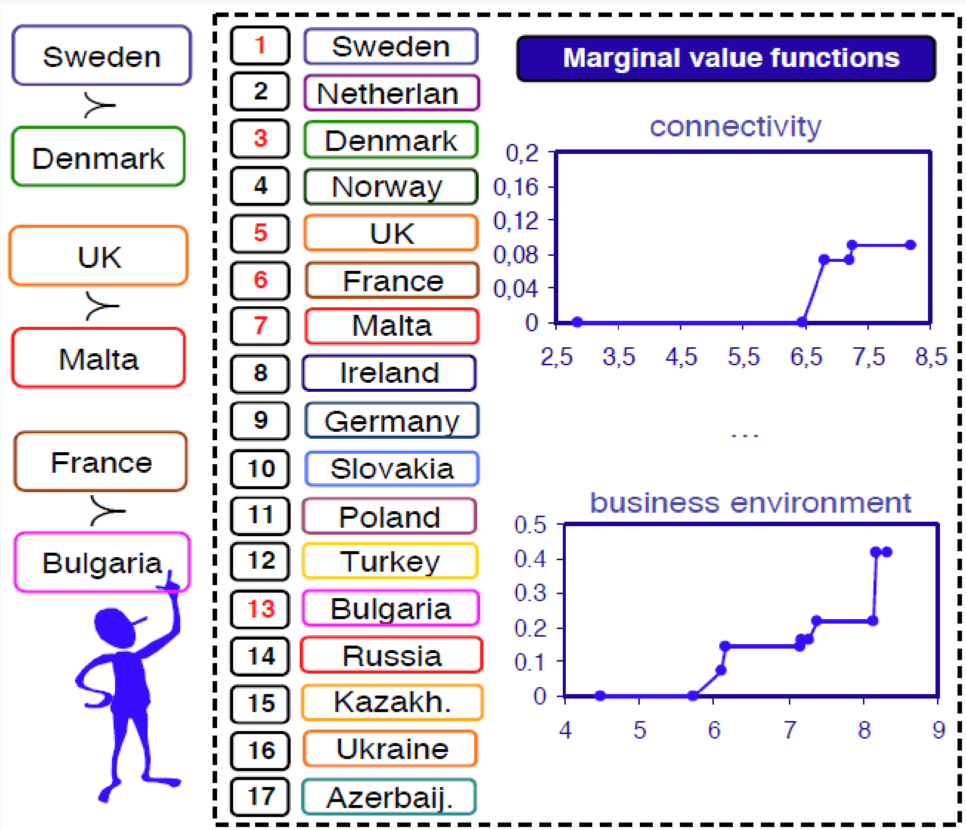
Example

- Ranking of countries wrt digital economy (quality of information and technology infrastructure) (Economist Intelligence Unit in 2010)

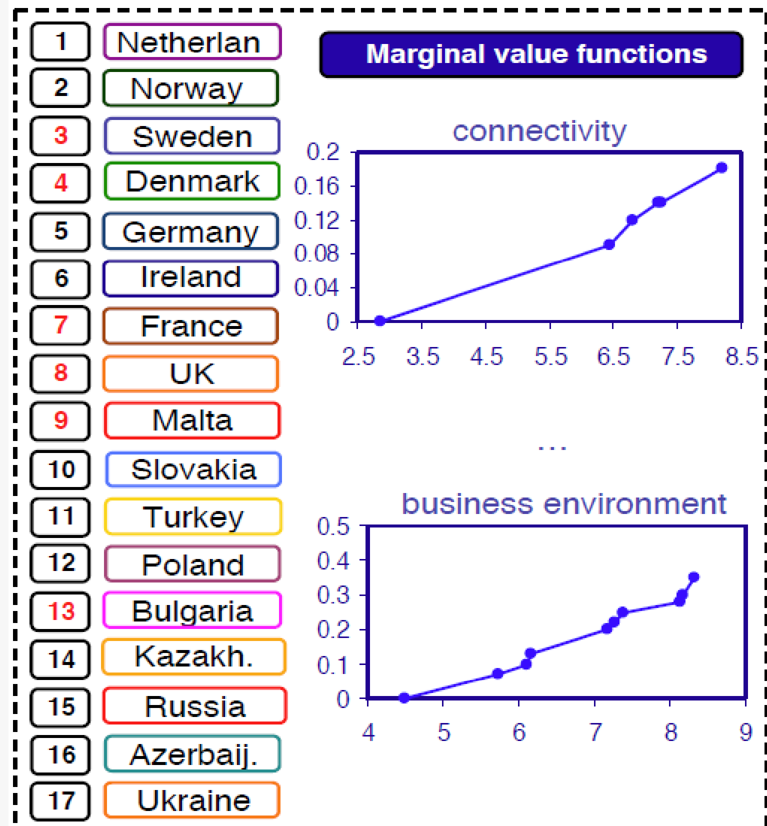


Value function reproducing pairwise comparisons is not unique

Compatible value function ranks all countries while respecting the preference information



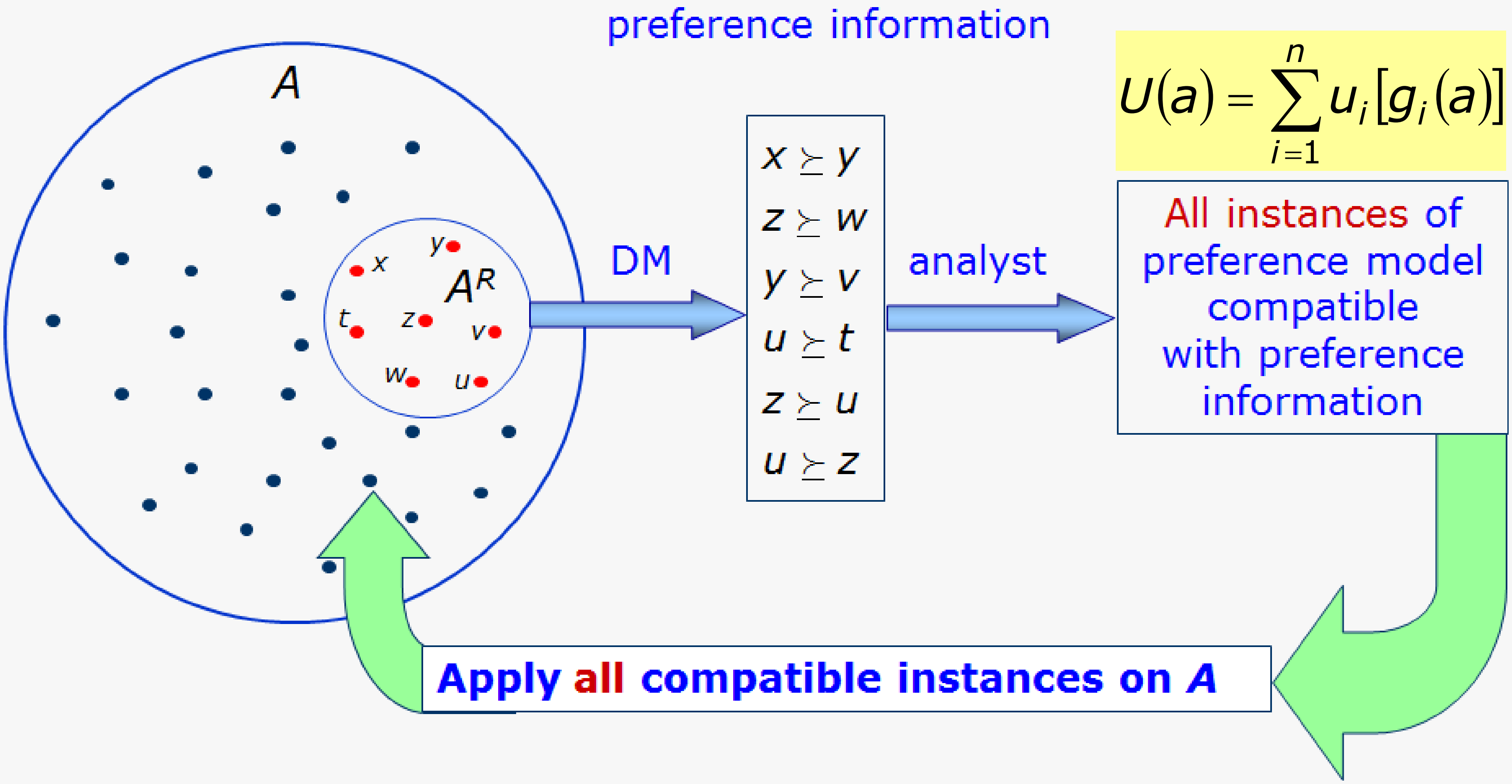
Another compatible value function may rank the countries otherwise



The two rankings are substantially different, although both reproduce the same preference information

Robust Ordinal Regression for value function preference model

Non-univocal representation - **R**obust **O**rdinal **R**egression - UTA^{GMS}



S. Greco, R. Słowiński, J. Figueira, V. Mousseau: Robust ordinal regression. Chapter 9 [in]: *Trends in Multiple Criteria Decision Analysis*. Springer, New York, 2010, pp. 241-283

ROR – possible and necessary preference relations

- The **possible** preference relation: for all alternatives $x,y \in A$,

  $x \succsim^P y \Leftrightarrow U(x) \geq U(y)$ **for at least one** compatible value function

  (complete and negatively transitive)

- The **necessary** preference relation: for all alternatives $x,y \in A$,

  $x \succsim^N y \Leftrightarrow U(x) \geq U(y)$ **for all** compatible value functions

  (partial preorder)

- When there is no preference information:

  necessary relation = dominance relation

$x \succsim^N y \;\Rightarrow\; x \succsim^P y$,

i.e., $\succsim^N \subseteq \succsim^P$

$x \succsim^N y$ or $y \succsim^P x$

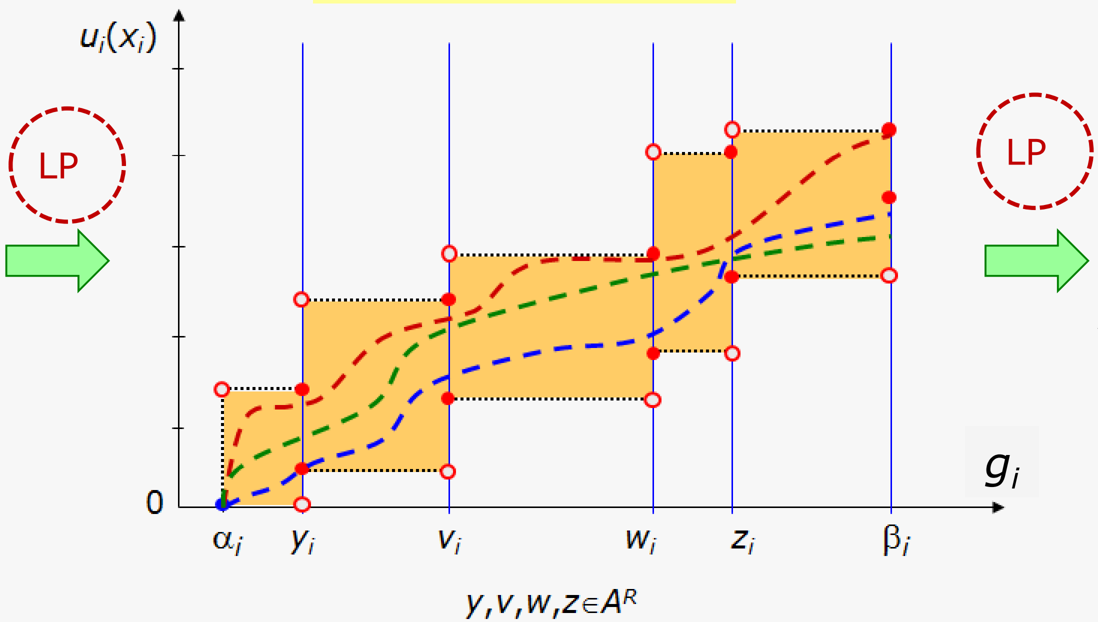
for all $x,y \in A$

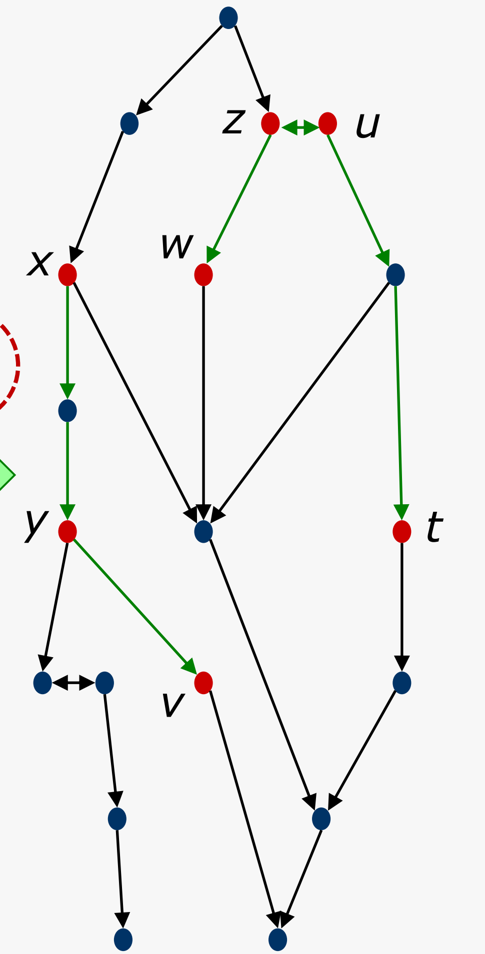
Non-univocal representation - **R**obust **O**rdinal **R**egression - UTA^{GMS}

$$U(a)=\sum_{i=1}^{n}u_i[g_i(a)]$$

preference information

$x \succsim y$
$z \succsim w$
$y \succsim v$
$u \succsim t$
$z \succsim u$
$u \succsim z$





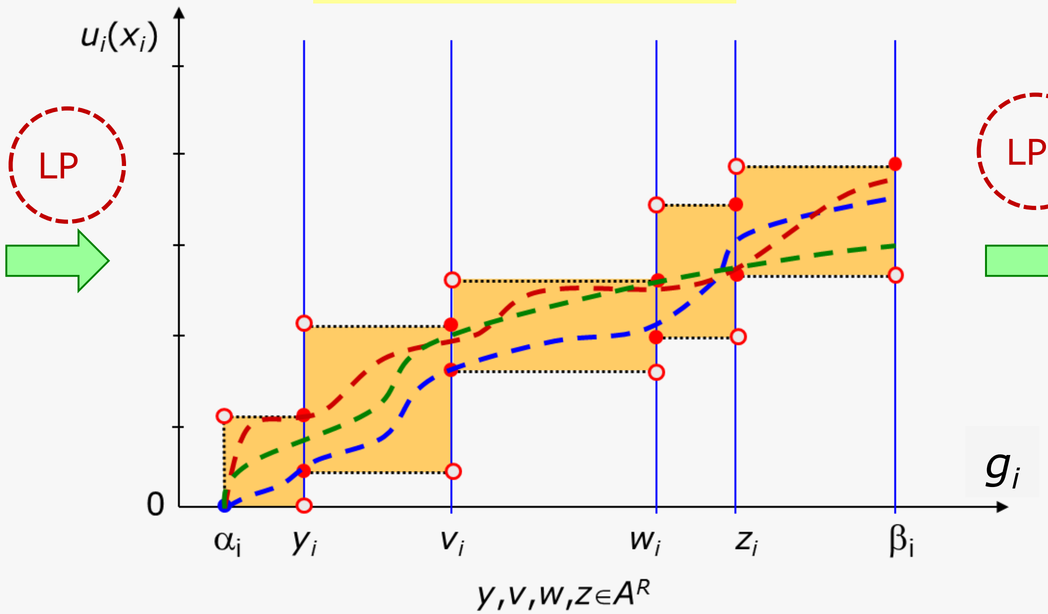
necessary ranking
(partial preorder)

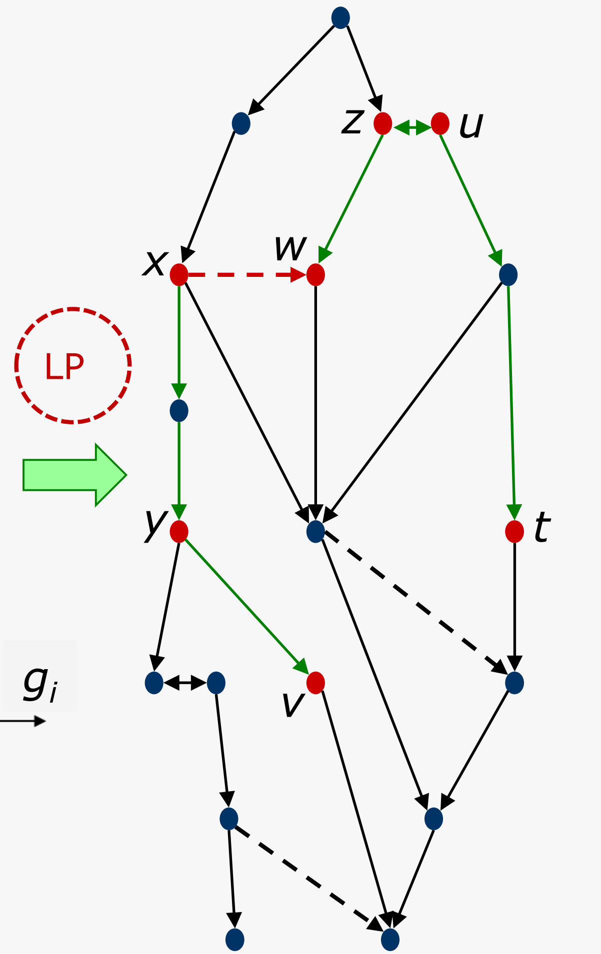
Non-univocal representation - **R**obust **O**rdinal **R**egression - UTA^{GMS}

$$U(a) = \sum_{i=1}^{n} u_i[g_i(a)]$$

additional preference information

$x \succsim y$
$z \succsim w$
$y \succsim v$
$u \succsim t$
$z \succsim u$
$u \succsim z$
$x \succsim w$

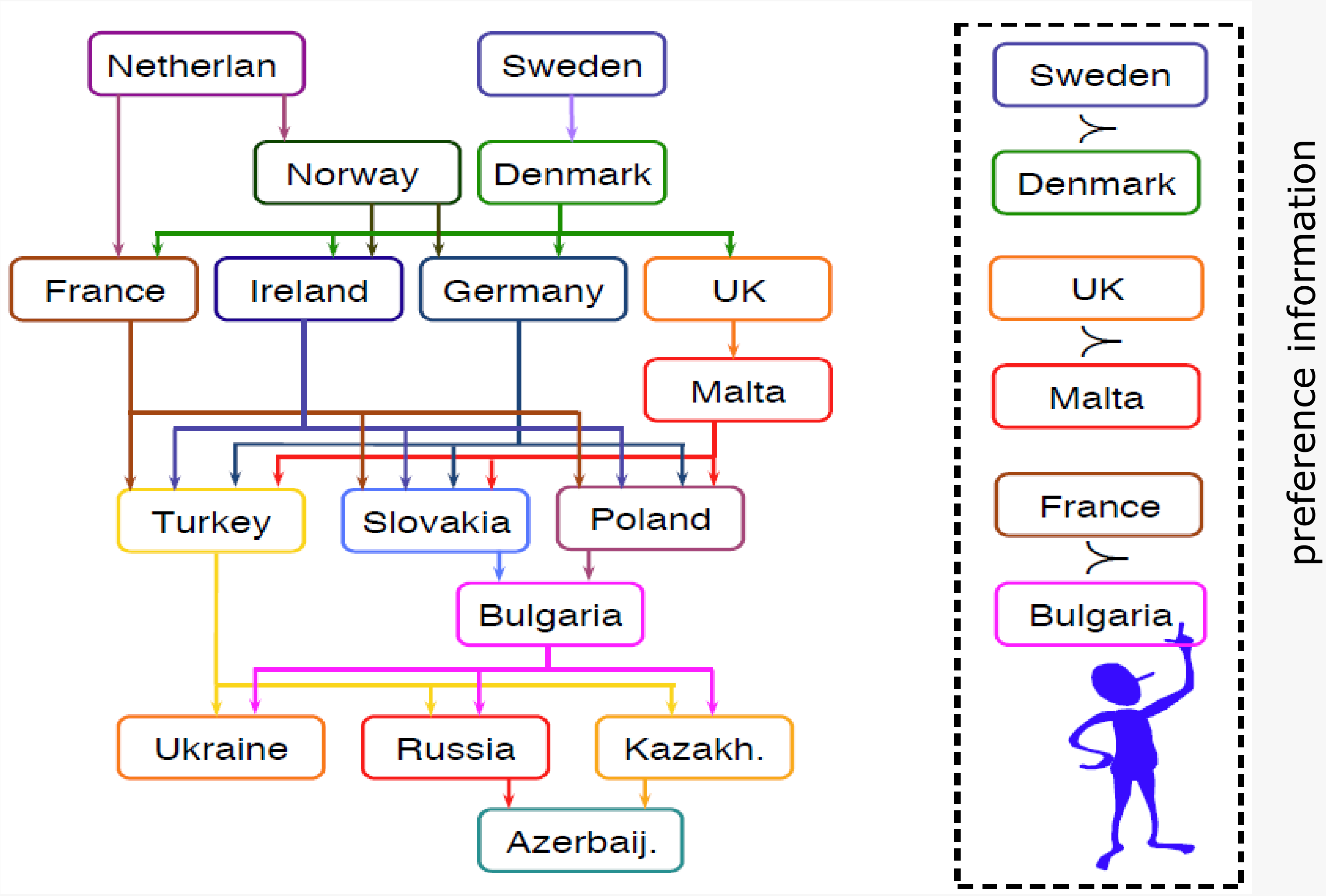




necessary ranking
enriched

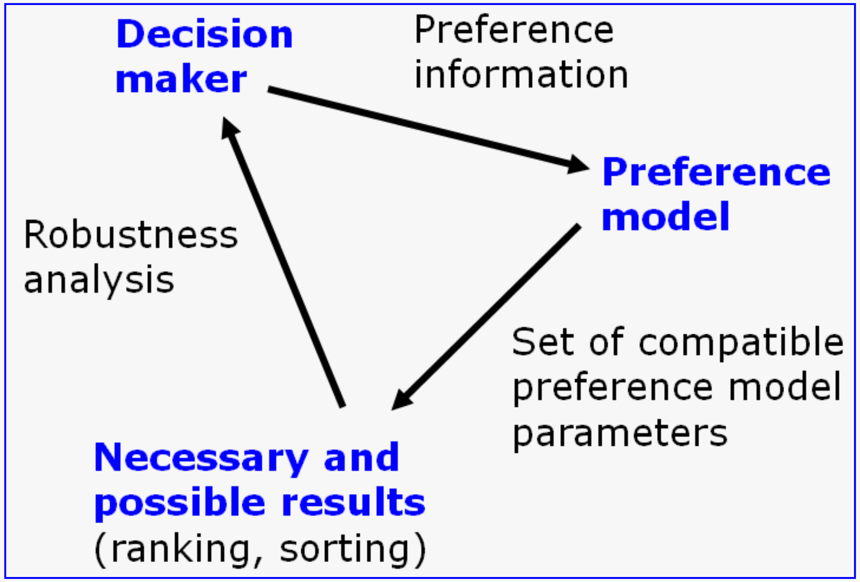
Recommendation in terms of a necessary ranking - UTA^{GMS}

- Necessary preference relation in the set of countries, obtained by all additive value functions compatible with preference information



Robust Ordinal Regression as a constructive learning

- Robust Ordinal Regression works in a loop with incremental elicitation of preferences → constructive learning
- Results are robust, because they take into account partial preference information



S. Corrente, S. Greco, M. Kadziński, R. Słowiński: Robust ordinal regression in preference learning and ranking. *Machine Learning*, 93 (2013) 381-422

Checking for the existence of a compatible value function

UTA^{GMS} method

$$\varepsilon^* = \max \varepsilon, \text{ subject to:}$$

$$E^{A^R}\left\{\begin{array}{l} \left.\begin{array}{ll} U(a^*) \geq U(b^*) + \varepsilon & \text{if } a^* \succ b^* \\ U(a^*) = U(b^*) & \text{if } a^* \sim b^* \end{array}\right\} \forall a^*, b^* \in A^R \\ u_i(x_i^k) - u_i(x_i^{k-1}) \geq 0, \quad i = 1, \ldots, n, \; k = 1, \ldots, m_i(A^R) \\ u_i(x_i^0) = 0, \quad i = 1, \ldots, n \\ \sum_{i=1}^{n} u_i(x_i^{m_i}) = 1 \end{array}\right.$$

If E^{A^R} is feasible and $\varepsilon^* > 0$, then there exists at least one value function compatible with the preference information

Calculating necessary and possible preference relations

- For all pairs of actions $a,b \in A$, their performances on criteria $g_i(a)$, $g_i(b)$ add to $m_i(A^R)$ characteristic points of marginal value function u_i , $i=1,\dots,n$; then E^{A^R} becomes $E(a,b)$
- Consider constraints:

$$\left.\begin{array}{c} U(b) \geq U(a) + \varepsilon \\ E(a,b) \end{array}\right\} E^N(a,b) \qquad \left.\begin{array}{c} U(a) \geq U(b) \\ E(a,b) \end{array}\right\} E^P(a,b)$$

- The necessary and the possible preference relations (LP problems):

$$a \succsim^N b \Leftrightarrow \text{if } E^N(a,b) \text{ infeasible or } \varepsilon^N(a,b) = \max \varepsilon, \text{ s.t. } E^N(a,b) \text{ is } \leq 0$$

$$a \succsim^P b \Leftrightarrow \text{if } E^P(a,b) \text{ feasible and } \varepsilon^P(a,b) = \max \varepsilon, \text{ s.t. } E^P(a,b) \text{ is } > 0$$

When the adopted value function fails to represent preferences…

If for a given preference information there is no compatible value function, the user can:

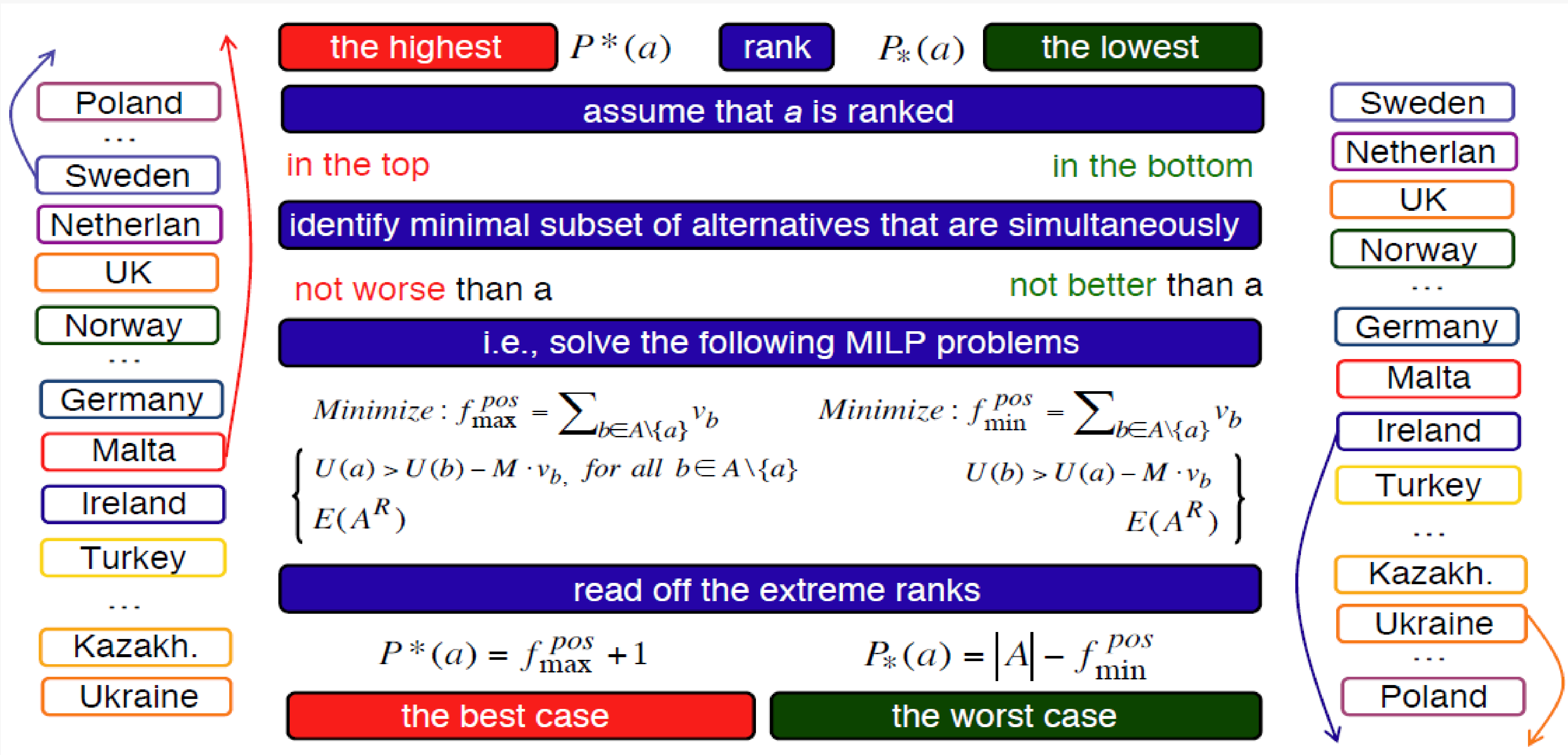
- identify and eliminate „troublesome” pieces of preference information (Mousseau et al. 2003),
- continue to use „not completely compatible” set of value functions with an acceptable approximation error
- augment the complexity of the value function, i.e., pass from additive value function to Choquet integral or augmented additive value function taking into account interactions between criteria

S. Greco, V. Mousseau, R. Słowiński: UTA^{GMS}–INT: robust ordinal regression of value functions handling interacting criteria. *EJOR*, 239 (2014) 711–730.

Extreme ranking analysis

Extreme ranking analysis

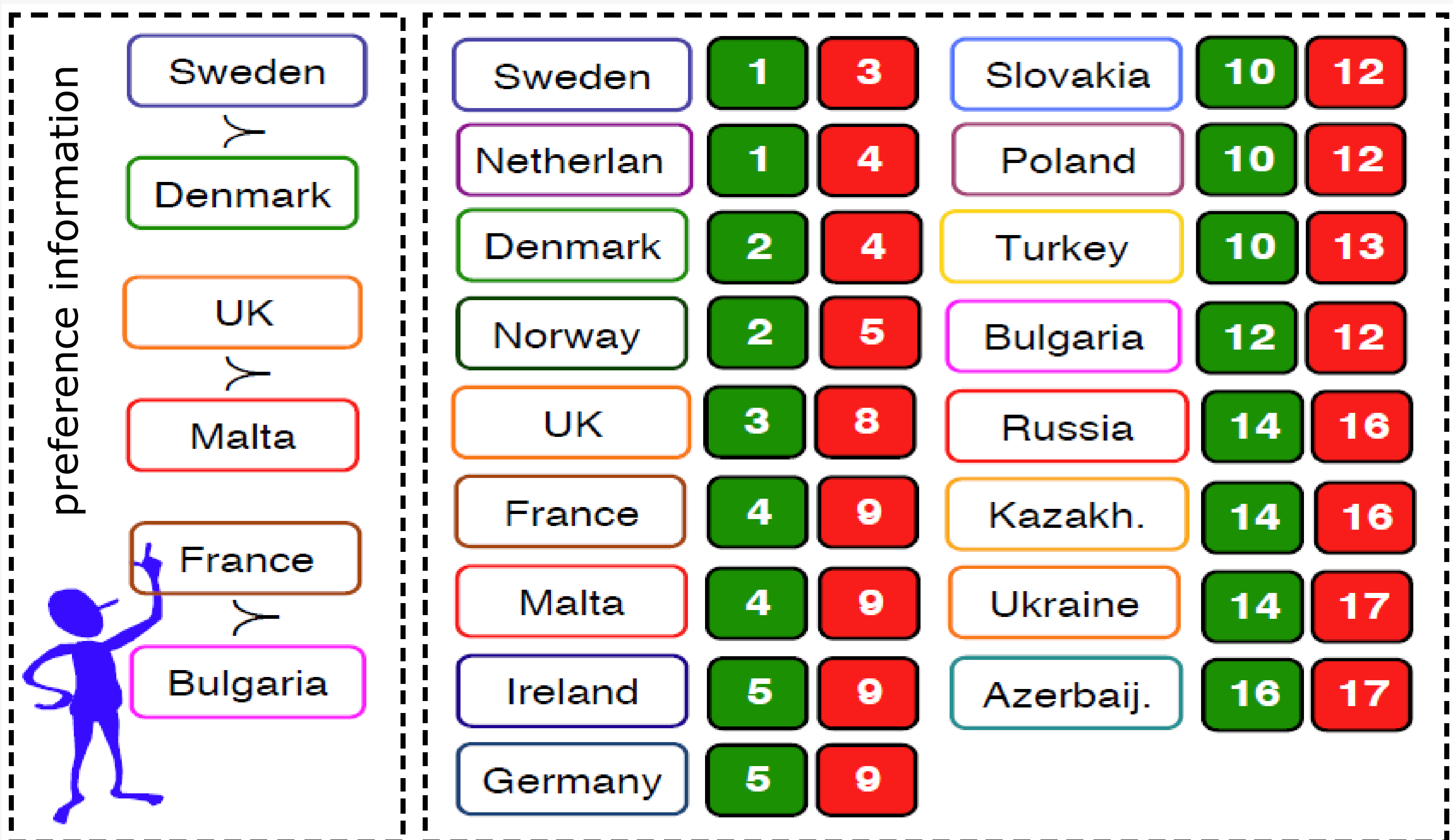
- Collate each action with all the remaining actions jointly
- Compute the highest and the lowest ranks and scores



M. Kadziński, S. Greco, R. Słowiński: Extreme ranking analysis in robust ordinal regression. *OMEGA*, 40 (2012) 488-501

Extreme ranking analysis

- Narrow ranges (Bulgaria) vs. wide ranges (UK)
- Interactive specification of new pairwise comparisons, e.g., (UK, Ireland), (Poland, Slovakia)
- Choice of the best actions, e.g., BEST = $\{a \in A: P^*(a)=1\}$

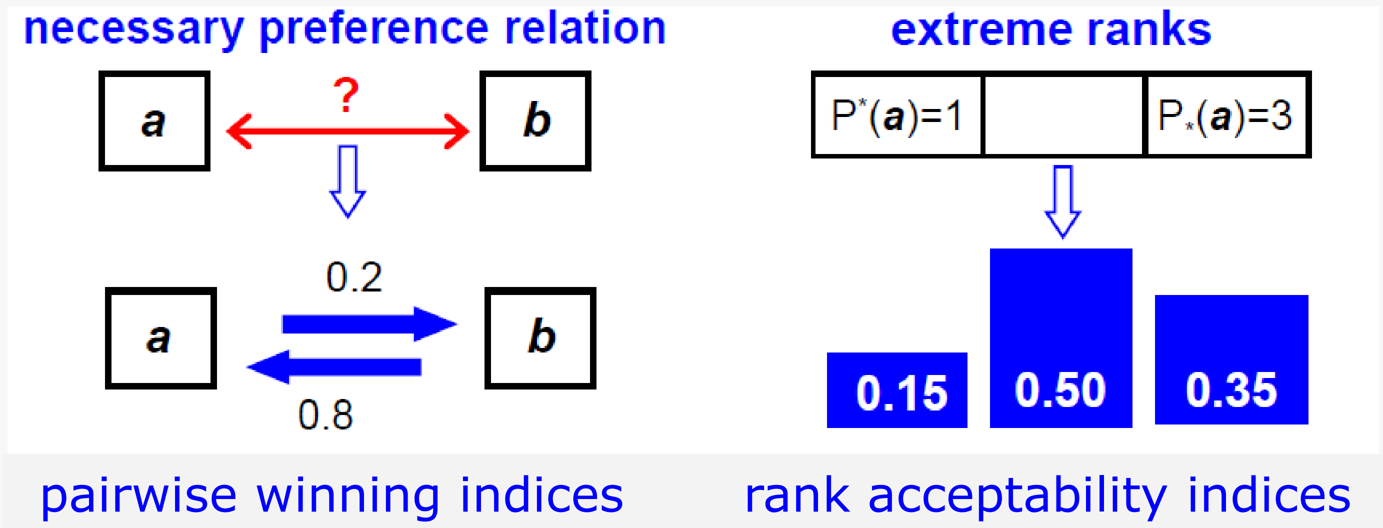


Stochastic ordinal regression

Stochastic Multiobjective Acceptability Analysis & ROR = SOR

- When the necessary preference relation $\succsim^N$ is poor, it leaves many pairs of alternatives incomparable, i.e., $a \not\succsim^P b$ and $b \not\succsim^P a$
- The number of compatible value functions constrained by available preference information is infinite
- One can sample these compatible value functions within the constraints and check the frequency with which:
  - $a \succ b$ – pairwise winning index $p(a,b)$,
  - a gets position i in the ranking – rank acceptability index b^i_a
- The sampling is performed using the *Hit and Run* algorithm (Smith 1984) (Monte Carlo simulation)

Stochastic Multiobjective Acceptability Analysis & ROR → SOR

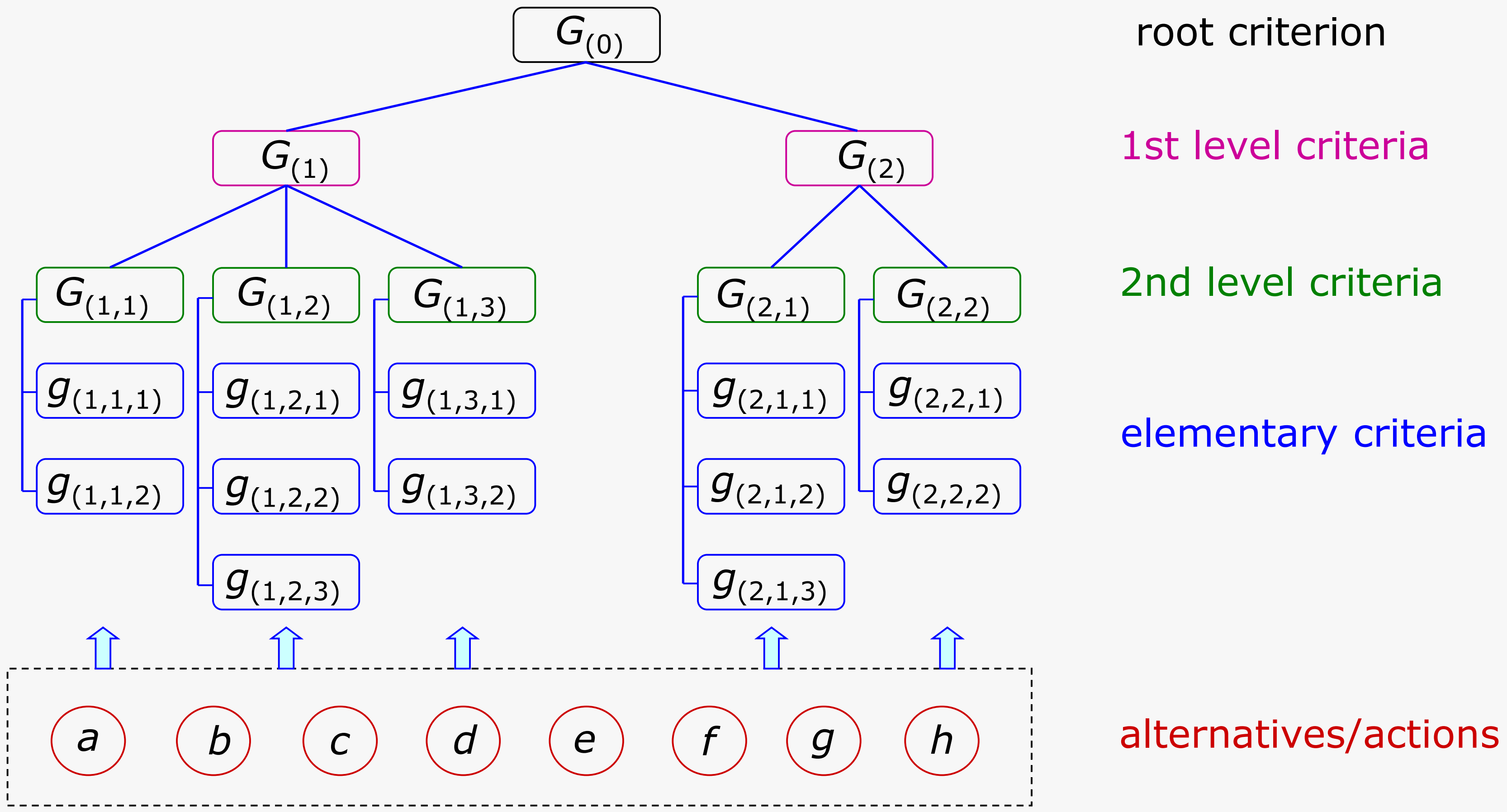


M. Kadziński, T. Tervonen, Stochastic ordinal regression for multiple criteria sorting, *Decision Support Systems*, 55(1), 55-66, 2013

S. Corrente, S. Greco, M. Kadziński, R. Słowiński: Inducing probability distributions on the set of value functions by Subjective Stochastic Ordinal Regression. *Knowledge Based Systems*, 112 (2016) 26–36

Robust Ordinal Regression for hierarchy of criteria

Multiple Criteria Hierarchy Process (MCHP)



S. Corrente, S. Greco, R. Słowiński: Multiple Criteria Hierarchy Process in Robust Ordinal Regression. *Decision Support Systems*, 53 (2012) 660-674

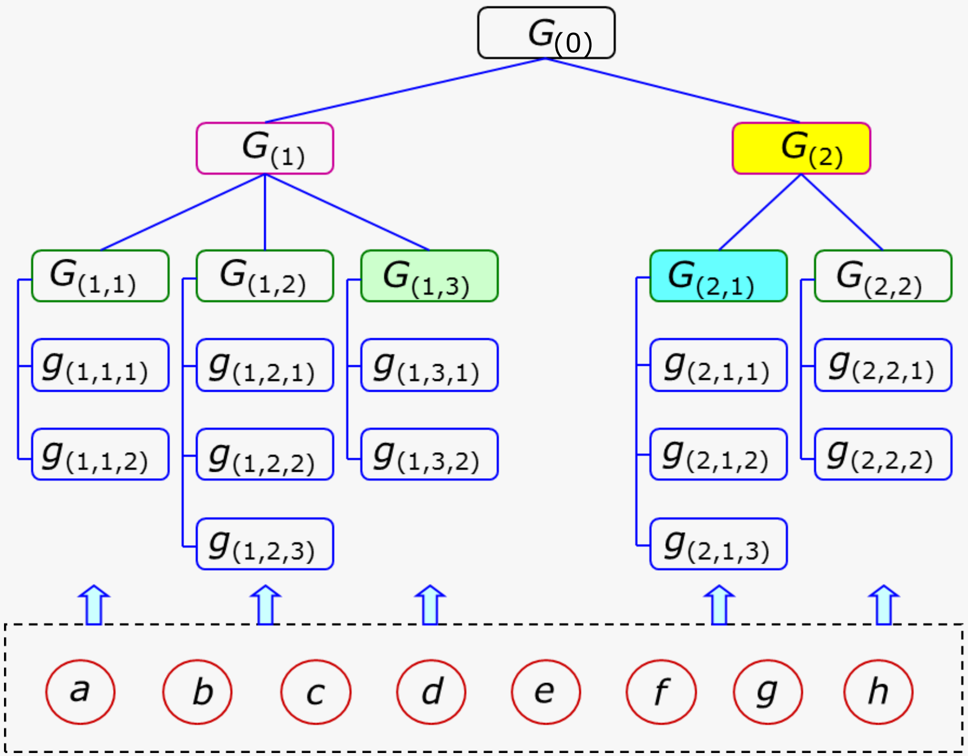
Multiple Criteria Hierarchy Process (MCHP) – main idea

- We wish to consider preference relation $\succeq_{\mathbf{r}}$ in each node of the hierarchy tree, e.g.:

$$a \succeq_{(2)} b \;\Leftrightarrow\; U_{(2)}(a) \geq U_{(2)}(b)$$

$$c \succeq_{(1,3)} d \;\Leftrightarrow\; U_{(1,3)}(c) \geq U_{(1,3)}(d)$$

$$e \succeq_{(2,1)} f \;\Leftrightarrow\; U_{(2,1)}(e) \geq U_{(2,1)}(f)$$



S. Corrente, S. Greco, R. Słowiński: Multiple Criteria Hierarchy Process in Robust Ordinal Regression. *Decision Support Systems*, 53 (2012) 660-674

MCHP with additive value function - preference elicitation & ROR

- Indirect preference information in particular nodes of the tree:
  - Pairwise comparison: a is at least as good as b on criterion $G_{\mathbf{r}}$

$$a \succsim_{\mathbf{r}} b \;\;\Leftrightarrow\;\; U_{\mathbf{r}}(a) \geq U_{\mathbf{r}}(b)$$

  - Intensity of preference: considering criterion $G_{\mathbf{r}}$ or $g_{\mathbf{t}}$,
    a is preferred to b at least as much as c is preferred to d

$$(a,b) \succsim^{*}_{\mathbf{r}} (c,d) \;\;\Leftrightarrow\;\; U_{\mathbf{r}}(a) - U_{\mathbf{r}}(b) \geq U_{\mathbf{r}}(c) - U_{\mathbf{r}}(d)$$

$$(a,b) \succsim^{*}_{\mathbf{t}} (c,d) \;\;\Leftrightarrow\;\; u_{\mathbf{t}}(a) - u_{\mathbf{t}}(b) \geq u_{\mathbf{t}}(c) - u_{\mathbf{t}}(d)$$

Properties of necessary and possible preference relations in node $\mathbf{r}$

- Given two alternatives $a,b \in A$, and any non-elementary criterion $G_{\mathbf{r}}$:

(*i*) $a \succsim^{N}_{(\mathbf{r},j)} b$ for all $j = 1,\ldots,n(\mathbf{r}) \;\; \Rightarrow \;\; a \succsim^{N}_{\mathbf{r}} b$

(*ii*) $a \succsim^{N}_{(\mathbf{r},j)} b$ for all $j = 1,\ldots,n(\mathbf{r}),\;\; j \neq w,$ and $a \succsim^{P}_{(\mathbf{r},w)} b \;\; \Rightarrow \;\; a \succsim^{P}_{\mathbf{r}} b$

(*iii*) $not\left(a \succsim^{P}_{(\mathbf{r},j)} b\right)$ for all $j = 1,\ldots,n(\mathbf{r}) \;\; \Rightarrow \;\; not\left(a \succsim^{P}_{\mathbf{r}} b\right)$

$\Updownarrow$

$a \succsim^{P}_{\mathbf{r}} b \;\; \Rightarrow a \succsim^{P}_{(\mathbf{r},j)} b$ for at least one $j \in \{1,\ldots,n(\mathbf{r})\}$

- Remark: hierarchical properties are expressed in terms of preference
  - necessary (*i*)
  - necessary & possible (*ii*)
  - possible (*iii*)

Multiple Criteria Hierarchy Process (MCHP) – value function & ROR

- Other developments in MCHP for value function and ROR:
  - Choquet integral value function
  - Choquet integral value function and Stochastic Ordinal Regression
  - MCHP for sorting problems with additive value functions

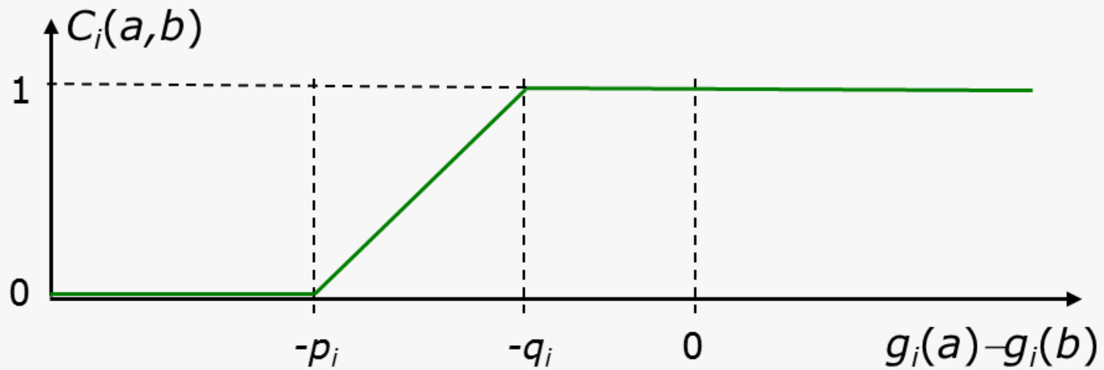
S. Angilella, S. Corrente, S. Greco, R. Słowiński: Robust Ordinal Regression and Stochastic Multiobjective Acceptability Analysis in Multiple Criteria Hierarchy Process for the Choquet integral preference model. *OMEGA*, 63 (2016) 154-169

Robust Ordinal Regression
for outranking relation preference model

Robust Ordinal Regression approach for outranking methods

- Concordance test: checks if the coalition of criteria concordant with the hypothesis *aSb* is strong enough:

$$C(a,b) = \frac{\sum_{i=1}^{n} w_i C_i(a,b)}{\sum_{i=1}^{n} w_i} \qquad a,b \in A, \quad w_i \text{ are weights of criteria}$$



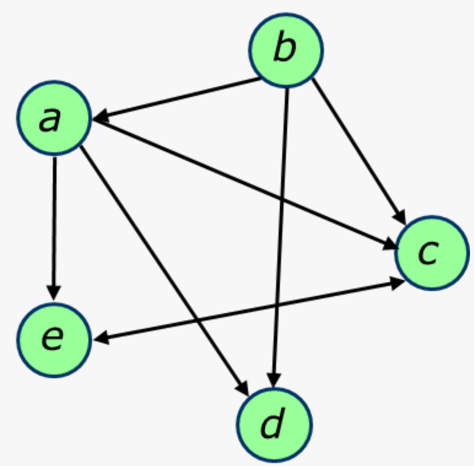
- Concordance test is positive if: $C(a,b) \geq \lambda$,
  where $\lambda \in [0.5, 1]$ is a cutting level (concordance threshold)

- No compensation between criteria <u>because the weights are not multiplied by performances</u> (weight w_i is a voting power of g_i)

Robust Ordinal Regression approach for outranking methods

- Discordance test: checks if among criteria discordant with the hypothesis *aSb* there is a strong opposition against *aSb*:
  - $g_i(b) - g_i(a) \geq v_i$ (for gain-type criterion)
  - $g_i(a) - g_i(b) \geq v_i$ (for cost-type criterion)
- Conclusion: *aSb* is true if and only if $C(a,b) \geq \lambda$ and there is no criterion strongly opposed (making veto) to the hypothesis
- For each couple $(a,b) \in A \times A$, one obtains relation S: true (1) or false (0)

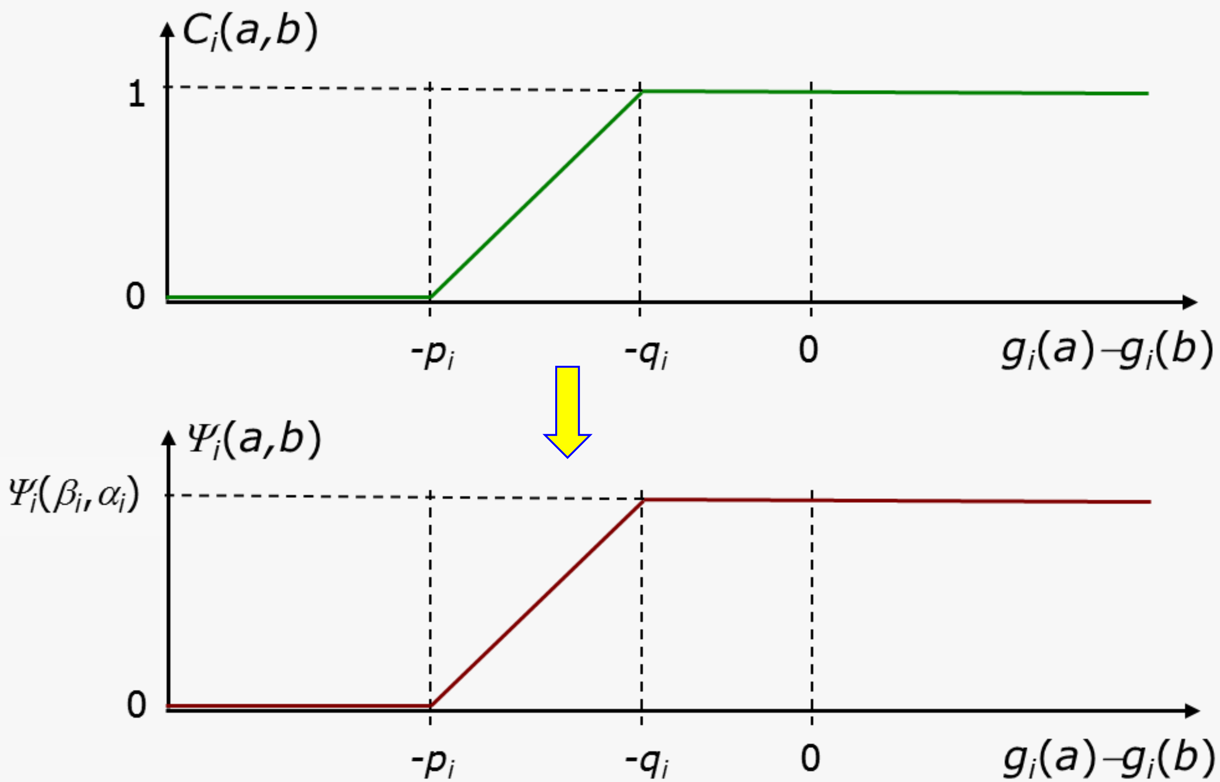
S	*a*	*b*	*c*	*d*	*e*
a	1	0	1	1	1
b	1	1	1	1	0
c	0	0	1	0	1
d	0	0	0	1	0
e	0	0	1	0	1



Robust Ordinal Regression approach for outranking methods

- Assuming $\sum_{i=1}^{n} w_i = 1$, we have $C(a,b) = \sum_{i=1}^{n} w_i C_i(a,b) = \sum_{i=1}^{n} \Psi_i(a,b)$

  where $\Psi_i(a,b)$ is a non-decreasing function of $g_i(a) - g_i(b)$



where α_i, β_i are, respectively, the worst and the best possible performance on criterion g_i, $i=1,\ldots,n$

Robust Ordinal Regression approach for outranking methods

- Preference information provided by the DM (ELECTRE^{GKMS}):

$$aSb \text{ or } aS^{c}b, \text{ for } a,b \in A^{R} \subset A$$

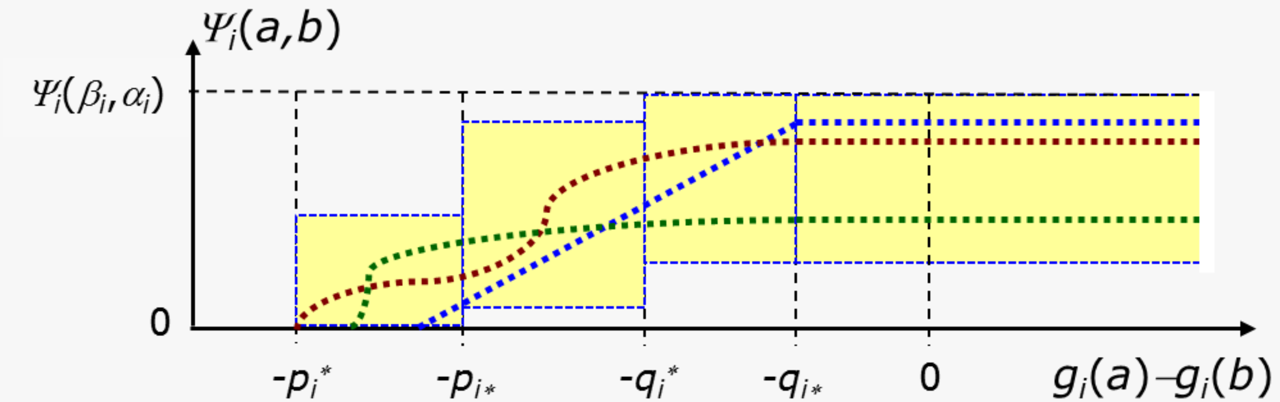
$[q_{i*}, q_i^*]$ - the range of indifference threshold allowed by the DM

$[p_{i*}, p_i^*]$ - the range of preference threshold allowed by the DM

S. Greco, M. Kadziński, V. Mousseau, R. Słowiński: ELECTRE^{GKMS}: Robust ordinal regression for outranking methods. *Decision Support Systems*, 52 (2011) 118-135

Robust Ordinal Regression approach for outranking methods

- Compatible outranking model is a set of marginal concordance functions $\Psi_i(a,b)$, cutting levels λ, indifference q_i, preference p_i, and veto thresholds v_i, $i=1,\dots,n$, reproducing the DM's preference information concerning pairs $(a,b)\in A^R\times A^R$



Robust ordinal regression approach for outranking methods

- Ordinal regression (compatibility) constraints E^{A^R}:

If aSb for $(a,b)\in A^R\times A^R$:

$$C(a,b)=\sum_{i=1}^{n}\Psi_i(a,b)\geq\lambda$$

$$g_i(b)-g_i(a)+\varepsilon\leq v_i,\ \ i=1,\ldots,n$$

aSb
concordance test (+)
and
discordance test (+)

If aS^cb for $(a,b)\in A^R\times A^R$:

$$C(a,b)=\sum_{i=1}^{n}\Psi_i(a,b)+\varepsilon\leq\lambda+M_0(a,b)$$

$$g_i(b)-g_i(a)\geq v_i-\delta M_i(a,b),\ \ i=1,\ldots,n$$

$$M_i(a,b)\in\{0,1\},\ \ i=0,1,\ldots,n$$

$$\sum_{i=0}^{n}M_i(a,b)\leq n,\ \ \text{where }\ \delta\ \text{ is a big given value}$$

aS^cb
concordance test (–)
or
discordance test (–)

$$0.5\leq\lambda\leq 1,$$

$$v_i\geq p_i^*+\varepsilon,\ \ \text{if}\ \left[p_{i*},p_i^*\right]\ \text{was given}$$

$v_i\geq g_i(b)-g_i(a)+\varepsilon,\ \ v_i\geq g_i(a)-g_i(b)+\varepsilon,$ if $a\sim_i b$ was given, $i\in\{1,\ldots,n\}$

Robust Ordinal Regression approach for outranking methods

- Given a pair of alternatives $a,b \in A$, a necessarily outranks b:

$$aS^N b \;\Leftrightarrow\; \varepsilon^* \le 0$$

where $\varepsilon^* = max\ \varepsilon$

$$\text{subject to:}$$

$$\left.\begin{array}{l} E^{A^R} \\ C(a,b) = \sum_{i=1}^{n} \Psi_i(a,b) + \varepsilon \le \lambda + M_0(a,b) \\ g_i(b) - g_i(a) \ge v_i - \delta M_i(a,b) \\ M_i(a,b) \in \{0,1\},\quad i = 1,\dots,n,\quad \sum_{i=0}^{n} M_i(a,b) \le n \end{array}\right\} E^N(a,b)$$

- If $\varepsilon^* \le 0$ and constraints $E^N(a,b)$ are infeasible,

  then a outranks b for all compatible outranking models

  ($aS^N b$ because $aS^{CN} b$ is not possible)

Exploitation of outranking relations S^N, S^{CN}, S^P, S^{CP} in set A

- Choice problem:

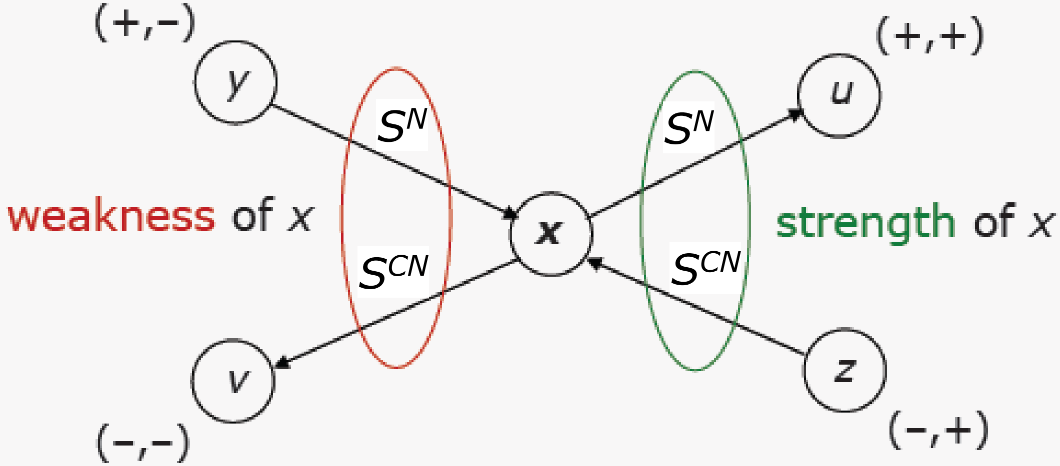
  Kernel of the necessary outranking graph S^N

- Ranking problem:

  Exploitation of the necessary outranking graph including S^N and S^{CN} using Net Flow Score procedure for each alternative $x \in A$:

  $$NFS(x) = strength(x) - weakness(x)$$

  S^N – positive argument, S^{CN} – negative argument



Ranking: complete preorder determined by $NFS(x)$ in A

Robust Ordinal Regression approach for outranking methods

Necessary outranking

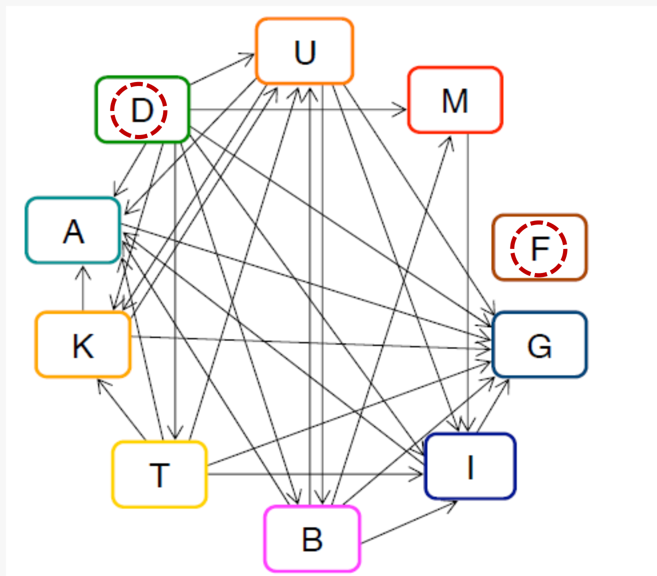
S^N	D	U	M	F	G	I	B	T	K	A
D	1	1	1	0	1	0	1	1	1	1
U	0	1	0	0	1	0	1	0	1	1
M	0	0	1	0	0	0	0	0	0	0
F	0	0	0	1	0	0	0	0	0	0
G	0	0	0	0	1	0	0	0	0	0
I	0	0	0	0	1	1	0	0	0	1
B	0	1	0	0	1	1	1	0	0	1
T	0	1	0	0	1	1	0	1	1	1
K	0	0	0	0	0	0	0	0	1	0
A	0	0	0	0	1	0	0	0	0	1

I S A
T S U
A S^C D
G S^C I

S^N	D	U	M	F	G	I	B	T	K	A
D	1	1	1	0	1	1	1	1	1	1
U	0	1	0	0	1	1	1	0	1	1
M	0	0	1	0	0	1	0	0	0	0
F	0	0	0	1	0	0	0	0	0	0
G	0	0	0	0	1	0	0	0	0	0
I	0	0	0	0	1	1	0	0	0	1
B	0	1	1	0	1	1	1	0	0	1
T	0	1	0	0	1	1	0	1	1	1
K	0	1	0	0	1	0	0	0	1	1
A	0	0	0	0	1	0	0	0	0	1

I S A
T S U
A S^C D
G S^C I
B S M
M S I
U S^C T

kernel



NFS ranking

D
T
F
B
U
K
I
A
M
G

Robust Ordinal Regression
for decision rule preference model

Syntax of monotonic decision rules

ordinal classification

- *if* $x_{q1} \succeq_{q1} r_{q1}$ *and* $x_{q2} \succeq_{q2} r_{q2}$ *and* … $x_{qp} \succeq_{qp} r_{qp}$, *then* $x \to$ class t or better
- *if* $x_{q1} \preceq_{q1} r_{q1}$ *and* $x_{q2} \preceq_{q2} r_{q2}$ *and* … $x_{qp} \preceq_{qp} r_{qp}$, *then* $x \to$ class t or worse

choice ranking cardinal criteria

- *if* $(x \succ_{q1}^{\geq h(q1)} y)$ *and* $(x \succ_{q2}^{\geq h(q2)} y)$ *and* … $(x \succ_{qp}^{\geq h(qp)} y)$, *then* xSy
- *if* $(x \succ_{q1}^{\leq h(q1)} y)$ *and* $(x \succ_{q2}^{\leq h(q2)} y)$ *and* … $(x \succ_{qp}^{\leq h(qp)} y)$, *then* $xS^{c}y$

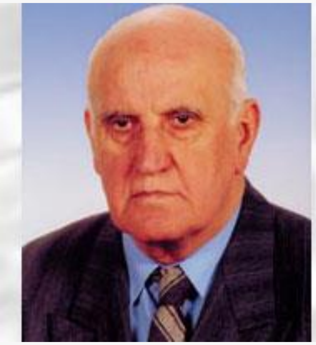
choice ranking ordinal criteria

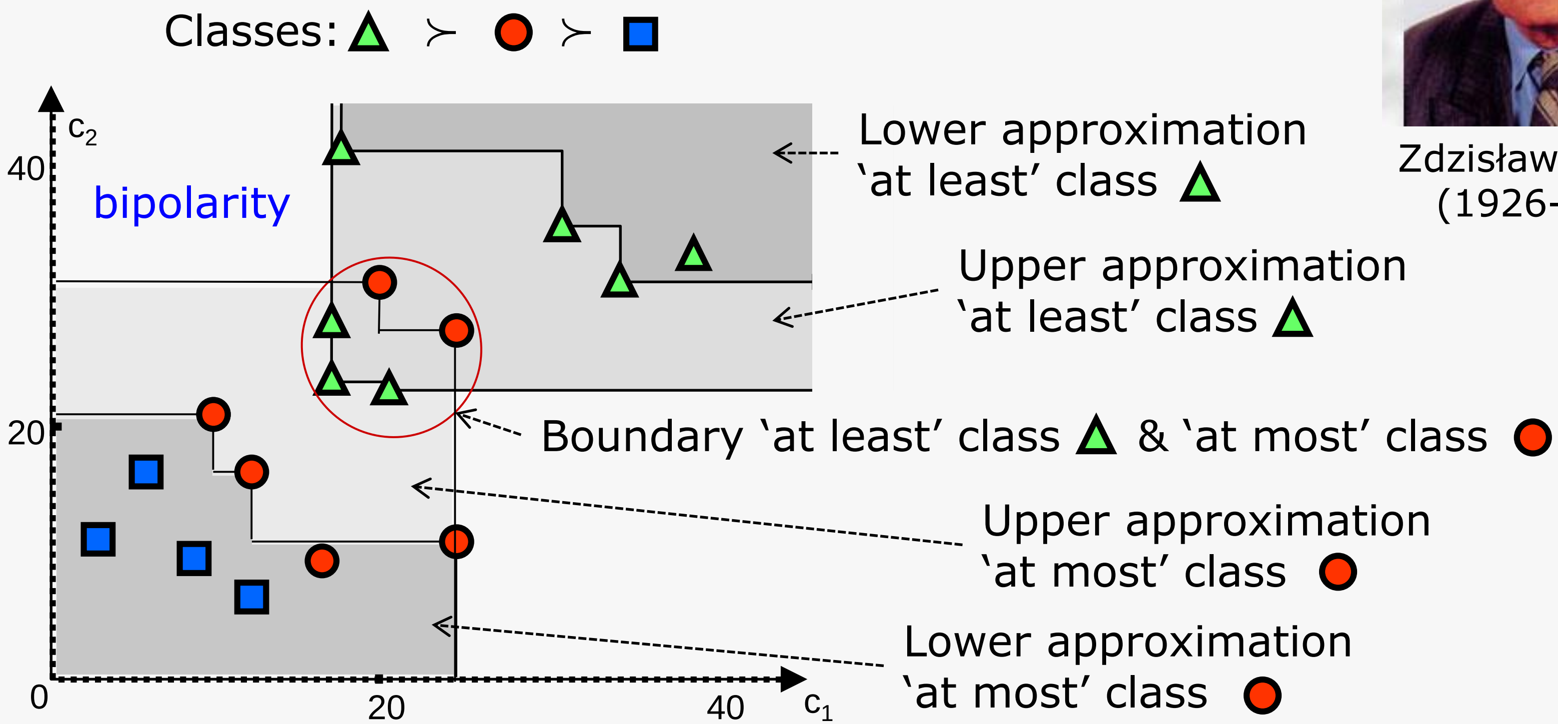
- *if* $x_{g1} \succeq_{g1} r_{q1}$ & $y_{g1} \preceq_{g1} r'_{q1}$ & … $x_{gp} \succeq_{gp} r_{gp}$ & $y_{gp} \preceq_{gp} r'_{gp}$, *then* xSy
- *if* $x_{g1} \preceq_{g1} r_{q1}$ & $y_{g1} \succeq_{g1} r'_{q1}$ & … $x_{gp} \preceq_{gp} r_{gp}$ & $y_{gp} \succeq_{gp} r'_{gp}$, *then* $xS^{c}y$

pair of objects x,y evaluated on criterion g_1

S.Greco, B.Matarazzo, R.Słowiński: Decision rule approach. Chapter 13 [in]: *Multiple Criteria Decision Analysis: State of the Art Surveys*, Springer, New York, 2016, pp. 497-552

Dominance-based Rough Set Approach (DRSA)





Zdzisław Pawlak
(1926-2006)

Dominance principle (comonotonicity)

If *x* is at least as good as *y* with respect to relevant **criteria**,

then *x* should be assigned to a class not worse than *y*

Z. Pawlak, Rough sets. *Int. J. of Computer & Information Sciences*, 11 (1982) 341-356

S. Greco, B. Matarazzo, R. Słowiński: Rough sets theory for multicriteria decision analysis. *EJOR*, 129 (2001) 1-47

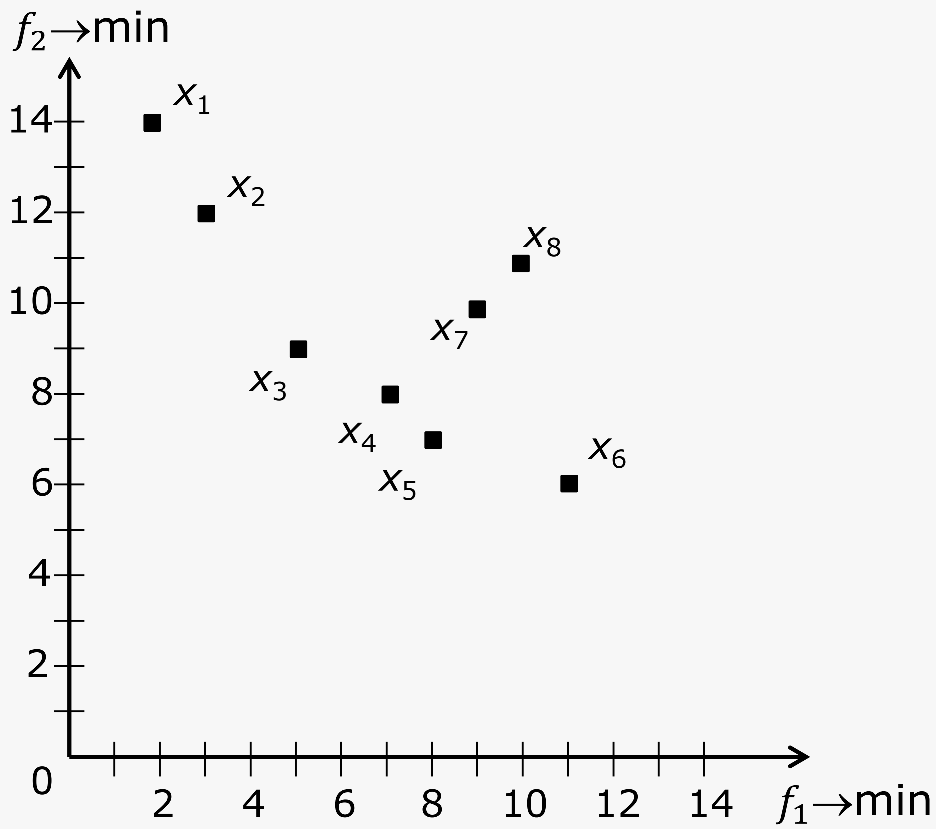
Preference modeling by dominance-based decision rules

- Dominance-based *„if…, then…”* decision rules are the only aggregation operators that:
  - give account of most complex interactions among attributes,
  - are non-compensatory,
  - accept ordinal evaluation scales and do not convert ordinal evaluations into cardinal ones,
- Rules identify values that drive DM’s decisions – each rule is a scenario of a causal relationship between evaluations on a subset of attributes and a comprehensive judgment

Example

Sample of 8 actions submitted to evaluation of the DM

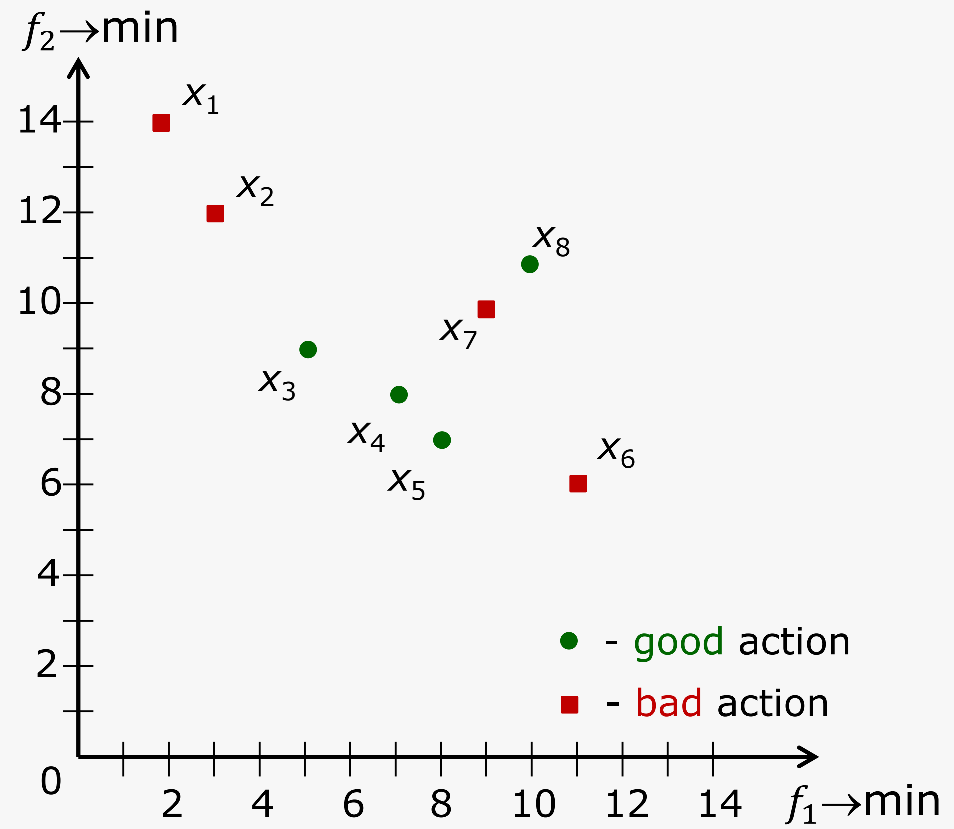
action	f_1	f_2	DM
x_1	2	14	
x_2	3	12	
x_3	5	9	
x_4	7	8	
x_5	8	7	
x_6	11	6	
x_7	9	10	
x_8	10	11	



Example

Sample of 8 actions – elicitation of preferences by the DM

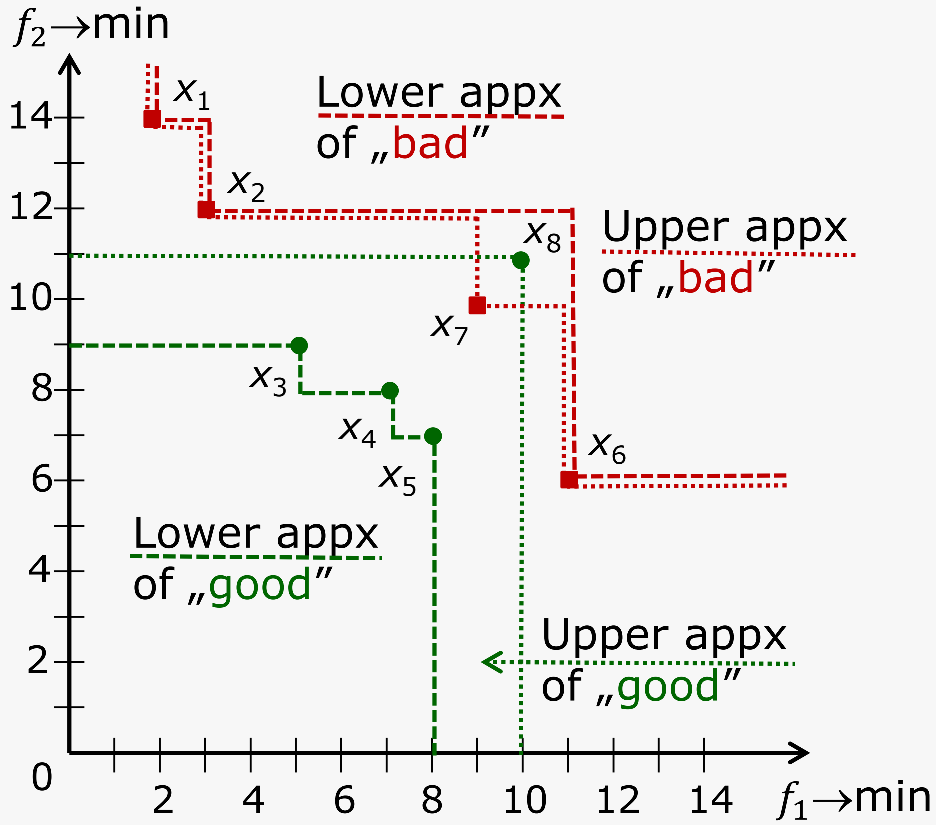
action	f_1	f_2	DM
x_1	2	14	bad
x_2	3	12	bad
x_3	5	9	good
x_4	7	8	good
x_5	8	7	good
x_6	11	6	bad
x_7	9	10	bad
x_8	10	11	good



Example

Sample of 8 actions – dominance-based rough approximations

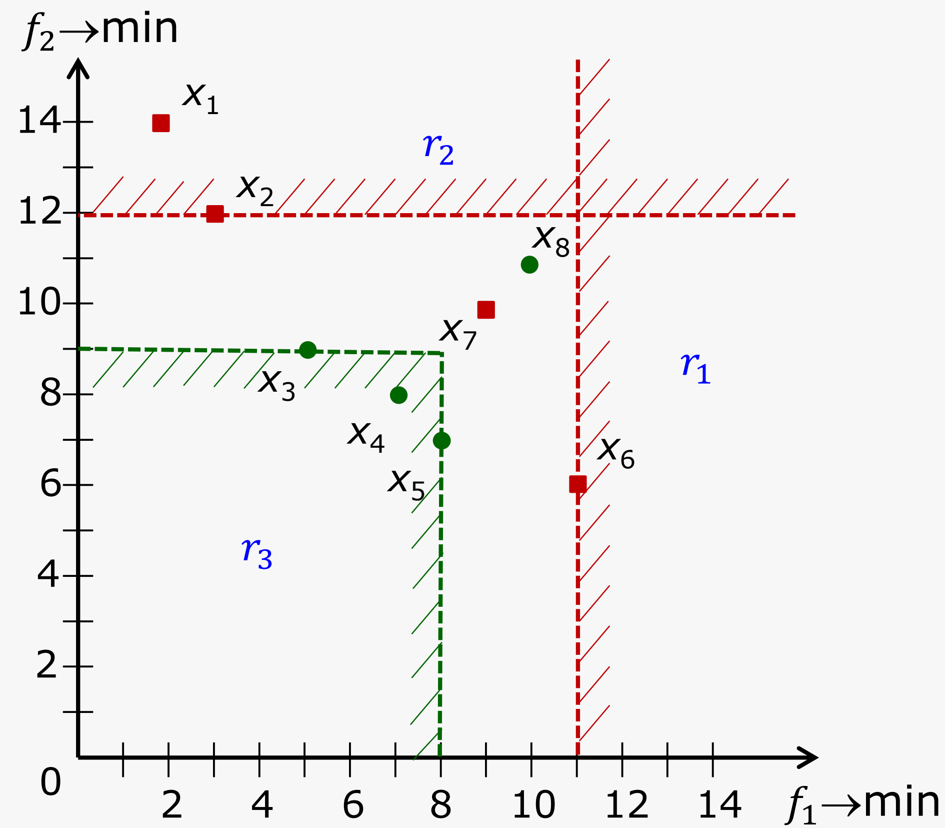
action	f_1	f_2	DM
x_1	2	14	bad
x_2	3	12	bad
x_3	5	9	good
x_4	7	8	good
x_5	8	7	good
x_6	11	6	bad
x_7	9	10	bad
x_8	10	11	good



Example

Sample of 8 actions – induction of certain decision rules

action	f_1	f_2	DM
x_1	2	14	bad
x_2	3	12	bad
x_3	5	9	good
x_4	7	8	good
x_5	8	7	good
x_6	11	6	bad
x_7	9	10	bad
x_8	10	11	good



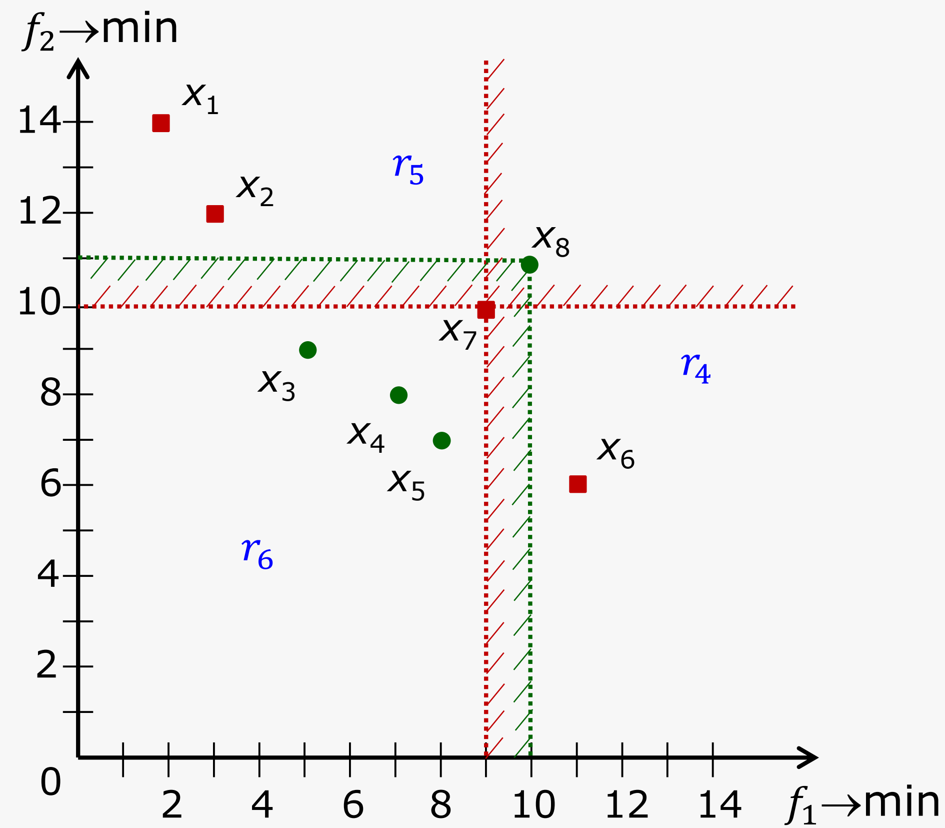
$D_\le$ $\begin{cases} r_1\text{: if } f_1(x)\ge 11, \text{ then } x \text{ is } \textit{certainly} \text{ bad} & \text{supported by } \{x_6\} \\ r_2\text{: if } f_2(x)\ge 12, \text{ then } x \text{ is } \textit{certainly} \text{ bad} & \text{supported by } \{x_1,x_2\} \end{cases}$

$D_\ge$ $\{$ r_3: if $f_1(x)\le 8$ & $f_2(x)\le 9$, then x is *certainly* good supported by $\{x_3,x_4,x_5\}$

Example

Sample of 8 actions – induction of possible decision rules

action	f_1	f_2	DM
x_1	2	14	bad
x_2	3	12	bad
x_3	5	9	good
x_4	7	8	good
x_5	8	7	good
x_6	11	6	bad
x_7	9	10	bad
x_8	10	11	good



$D_{\leq}$ $\begin{cases} r_4\text{: if } f_1(x)\geq 9, \text{ then } x \text{ is } \textit{possibly} \text{ bad} \\ r_5\text{: if } f_2(x)\geq 10, \text{ then } x \text{ is } \textit{possibly} \text{ bad} \end{cases}$

supported by $\{x_6,x_7,x_8\}$

supported by $\{x_1,x_2,x_7,x_8\}$

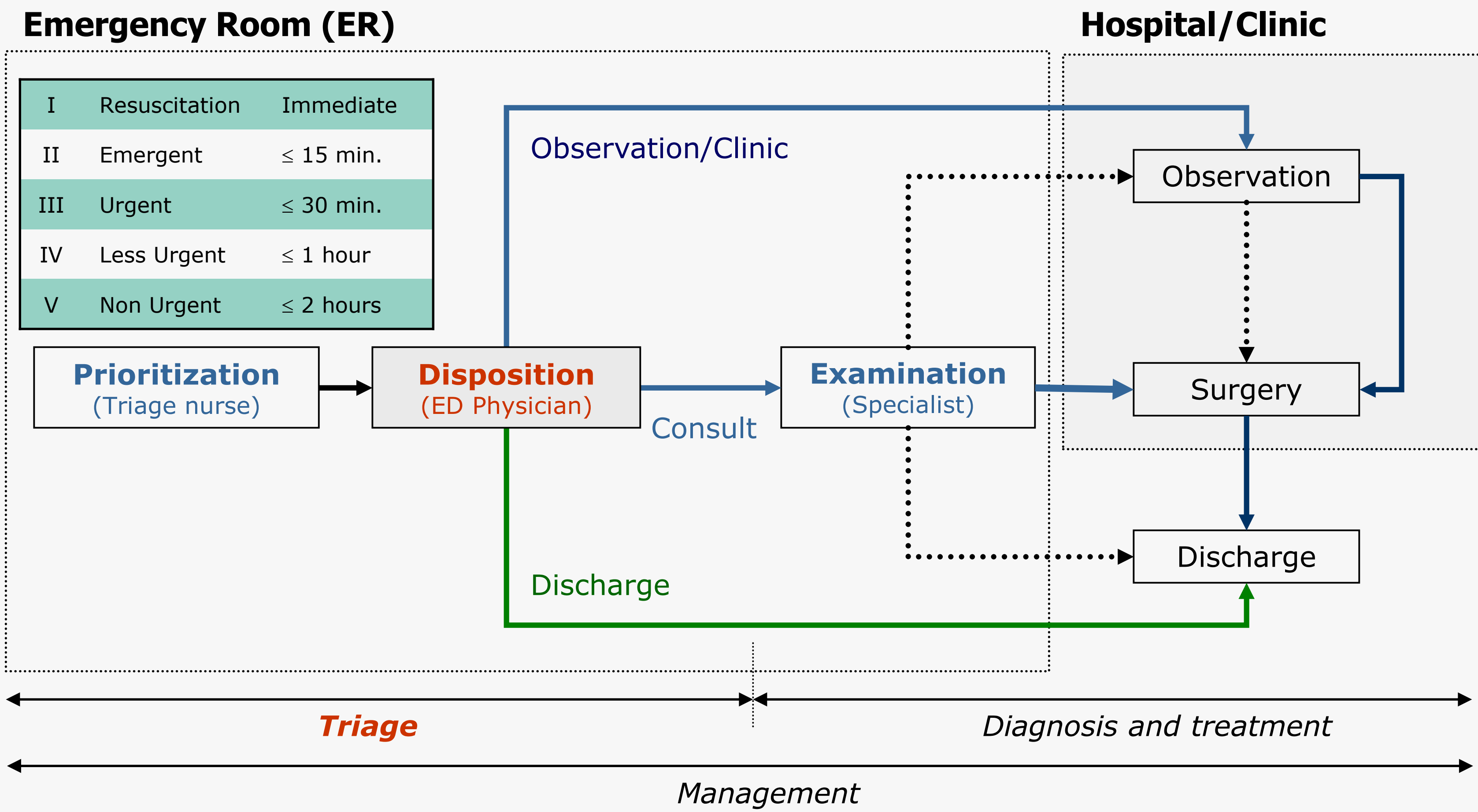
$D_{\geq}$ { r_6: if $f_1(x)\leq 10$ & $f_2(x)\leq 11$, then x is *possibly* good — supported by $\{x_3,x_4,x_5,x_7,x_8\}$

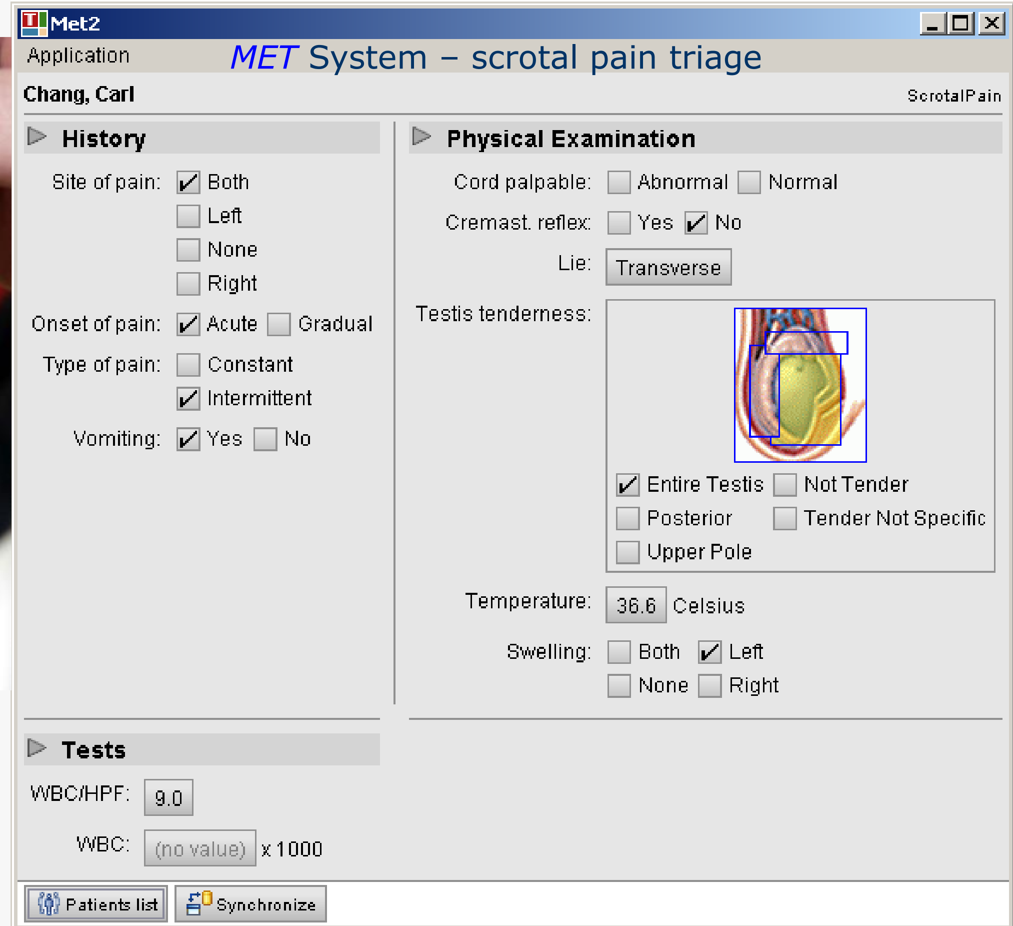
Examples of applications

- Total pediatric population >400,000
- 55,000 patient visits in the ER per year
- 3 pediatric general surgeons (supported by emergency physicians and residents)

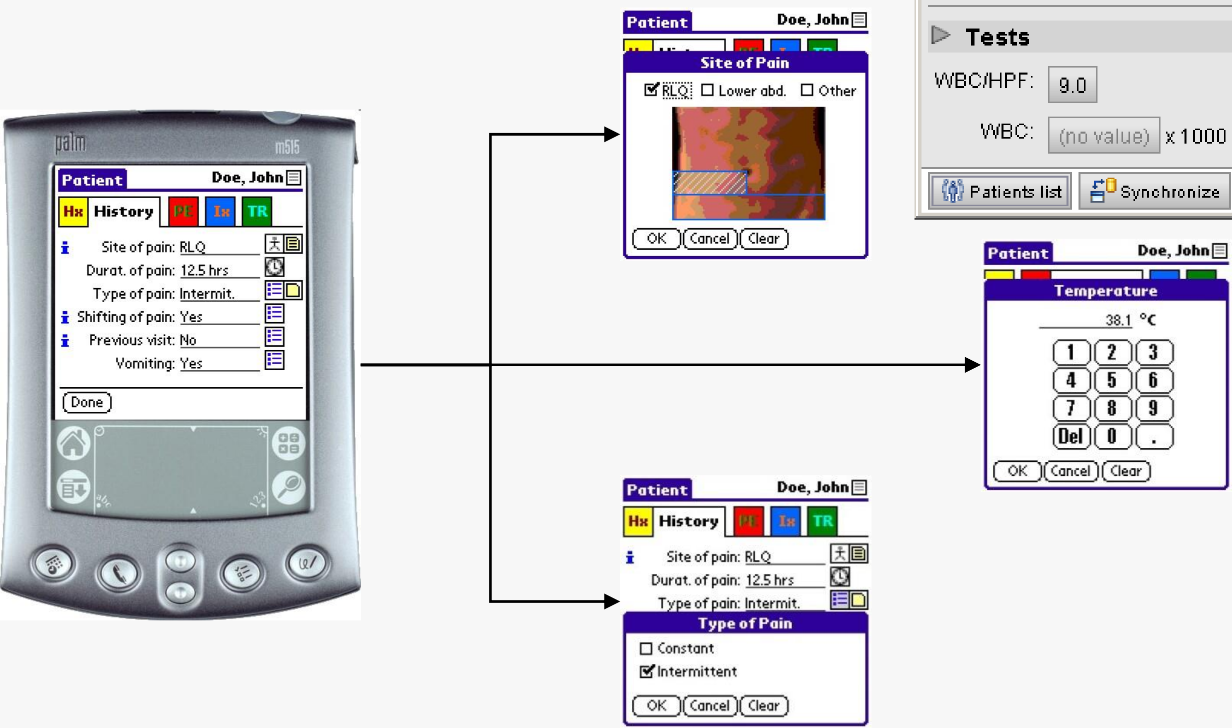
Triage Process

I	Resuscitation	Immediate
II	Emergent	≤ 15 min.
III	Urgent	≤ 30 min.
IV	Less Urgent	≤ 1 hour
V	Non Urgent	≤ 2 hours

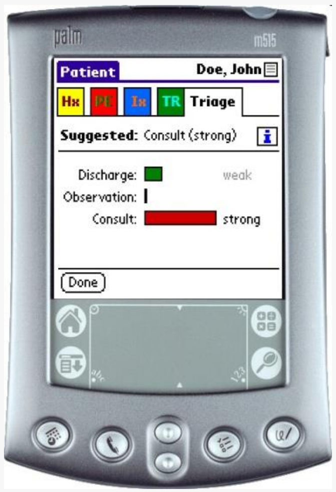




Met2
Application
MET System – scrotal pain triage
Chang, Carl
ScrotalPain
History
Site of pain: Both Left None Right
Onset of pain: Acute Gradual
Type of pain: Constant Intermittent
Vomiting: Yes No
Physical Examination
Cord palpable: Abnormal Normal
Cremast. reflex: Yes No
Lie: Transverse
Testis tenderness:
Entire Testis Not Tender
Posterior Tender Not Specific
Upper Pole
Temperature: 36.6 Celsius
Swelling: Both Left
None Right
Tests
WBC/HPF: 9.0
WBC: (no value) x 1000
Patients list
Synchronize



Patient Doe, John
Hx History PE Ix TR
Site of pain: RLQ
Durat. of pain: 12.5 hrs
Type of pain: Intermit.
Shifting of pain: Yes
Previous visit: No
Vomiting: Yes
Done
Site of Pain
RLQ Lower abd. Other
OK Cancel Clear
Temperature
38.1 °C
1 2 3 4 5 6 7 8 9 Del 0 .
OK Cancel Clear
Type of Pain
Constant
Intermittent
OK Cancel Clear



Patient Doe, John
Hx PE Ix TR Triage
Suggested: Consult (strong)
Discharge: weak
Observation:
Consult: strong
Done

Auto loan fraud detection using dominance-based rough set approach

- Bank data: 26 187 observations including 405 fraud events
- Accuracy of models compared:

Model	Class `fraud` detection rate [%]	Class `non-fraud` detection rate [%]	G-mean [%]
DRSA-BRE	79.09	82.56	80.81
Random Forest	18.19	99.98	42.62
SVM	13.9	99.94	37.26

- Examples of meaningful rules:

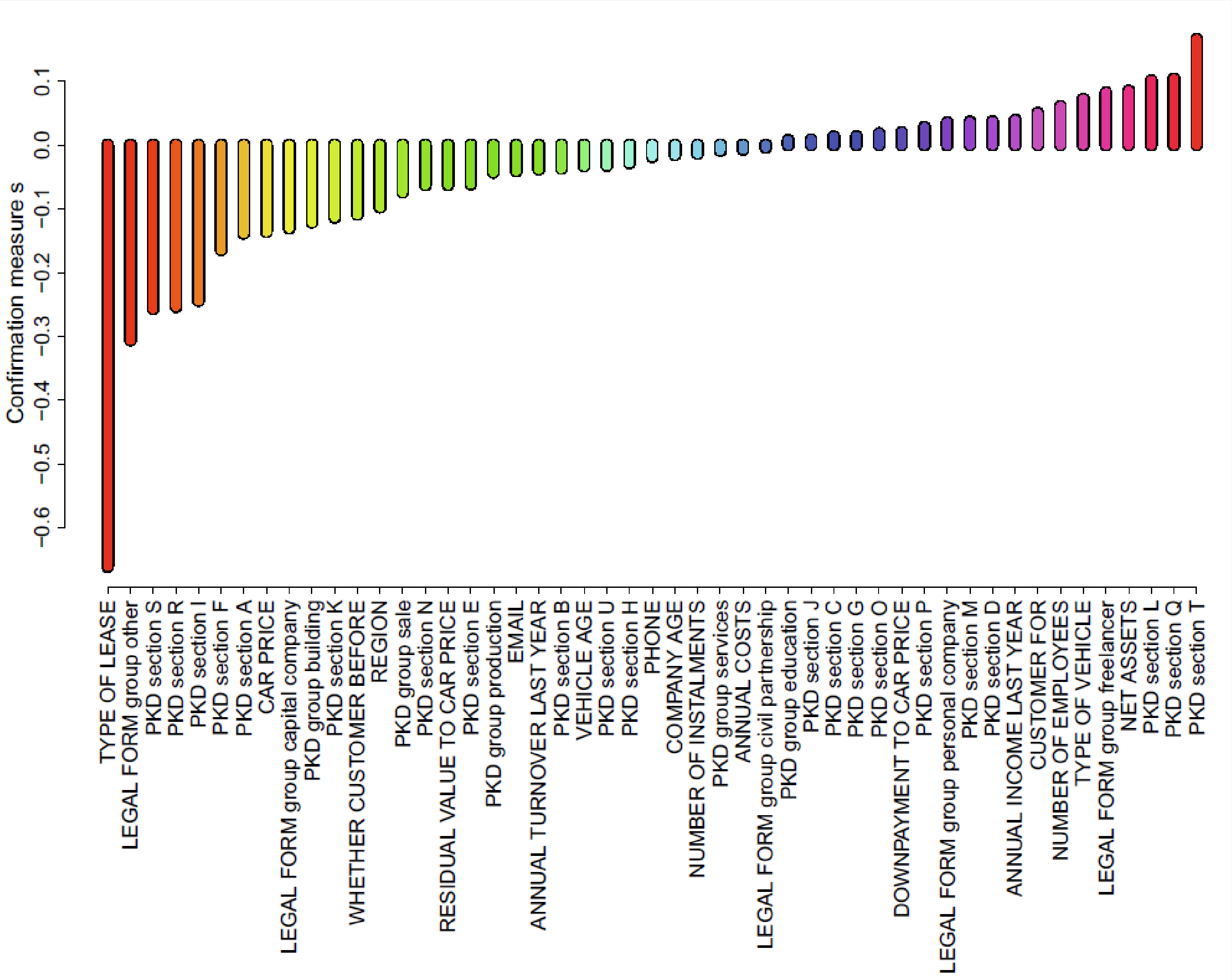
#1: if (NUMBER OF INSTALMENTS ≥ 60) and (CAR PRICE ≥ 55320) and (DOWNPAYMENT TO CAR PRICE ≤ 0.1) and (ANNUAL TURNOVER LAST YEAR ≥ 198000) and (COMPANY AGE ≤ 2), then fraud

#10: if (DOWNPAYMENT TO CAR PRICE ≤ 0.1) and (LEGAL FORM group = capital company) and (COMPANY AGE ≤ 6) and (PKD group = building), then fraud

J. Błaszczyński, A.T. de Almeida Filho, A. Matuszyk, M. Szeląg, R. Słowiński: Auto loan fraud detection using dominance-based rough set approach versus machine learning methods. *Expert Systems with Applications*, 163 (2021) 113740

Auto loan fraud detection using dominance-based rough set approach

- Importance of attributes in terms of attribute Bayesian confirmation:



Customer churn prediction using monotonic rules

- VC-DRSA performing sequential covering adapted to missing values was applied on a set of 10 000 customers (7963 exited, 2037 loyal)

Table 2: Comparison of avg. classification accuracy (%) in 10 × 10-fold cross-validation

%mv	ϵ-$\mathbb{D}_{1.5}^{mv}$	ϵ-$\mathbb{D}_{2}^{mv}$	C4.5	NB	SVM	RF	MP	RIPP	OLM	OSDL	MoNGEL
0	75.89	75.89	75.39	75.98	70.01	77.05	75.86	76.52	57.38	73.74	69.79
5	75.01	74.52	75.47	75.57	69.49	76.05	74.09	74.52	53.41	71.63	68.78
10	73.95	73.39	74.90	74.49	68.20	74.75	72.96	72.55	51.04	70.15	66.11
15	73.03	71.74	73.54	74.08	68.03	74.27	71.85	70.17	50.24	68.79	65.19
20	72.15	70.98	72.93	73.72	66.84	74.04	70.73	69.93	50.18	67.82	64.10
25	70.72	69.92	72.09	72.50	66.02	72.92	69.24	69.14	50.00	66.56	62.26

Table 7: Top rules induced by ϵ-$\mathbb{VC}$-$\mathbb{DRSA}$

ID	**Conditions**	**Decision**	ϵ	**Support**
98	CreditScore $\leq$ 712, Age $\geq$ 51, IsActiveMember $\leq$ 0	$Exited = 1$	0.005	264
91	NumOfProducts_c $\geq$ 3, CreditScore $\leq$ 789, Age $\geq$ 35	$Exited = 1$	0.003	221
108	Age $\geq$ 49, IsActiveMember $\leq$ 0, CreditScore $\leq$ 657, HasCrCard = 1	$Exited = 1$	0.005	172
111	Age $\geq$ 46, IsActiveMember $\leq$ 0, Geography = Germany, NumOfProducts_g $\leq$ 1, CreditScore $\leq$ 805	$Exited = 1$	0.003	171
97	Age $\geq$ 54, IsActiveMember $\leq$ 0, EstimatedSalary $\leq$ 123646.57	$Exited = 1$	0.002	155

M. Szeląg, R. Słowiński, Customer churn analytics using monotonic rules. *Proc. PP-RAI'2023*, Łódź 2023

Multiobjective Optimization

$$\begin{bmatrix} f_1(\boldsymbol{x}) \\ \vdots \\ f_n(\boldsymbol{x}) \end{bmatrix} \rightarrow \text{Min (or Max)}$$

subject to the constraints :

$$g_1(\boldsymbol{x}) \{\leq, =, \geq\} b_1$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$g_m(\boldsymbol{x}) \{\leq, =, \geq\} b_m$$

where $\boldsymbol{x}=[x_1,\ldots,x_k]$ - vector of decision variables (continuous/integer)

$f_j(x)$, $j=1,\ldots,n$ - real-valued objective functions

$g_i(x)$, $i=1,\ldots,m$ - real-valued functions of the constraints

b_i, $i=1,\ldots,m$ - constant RHS of the constraints

90

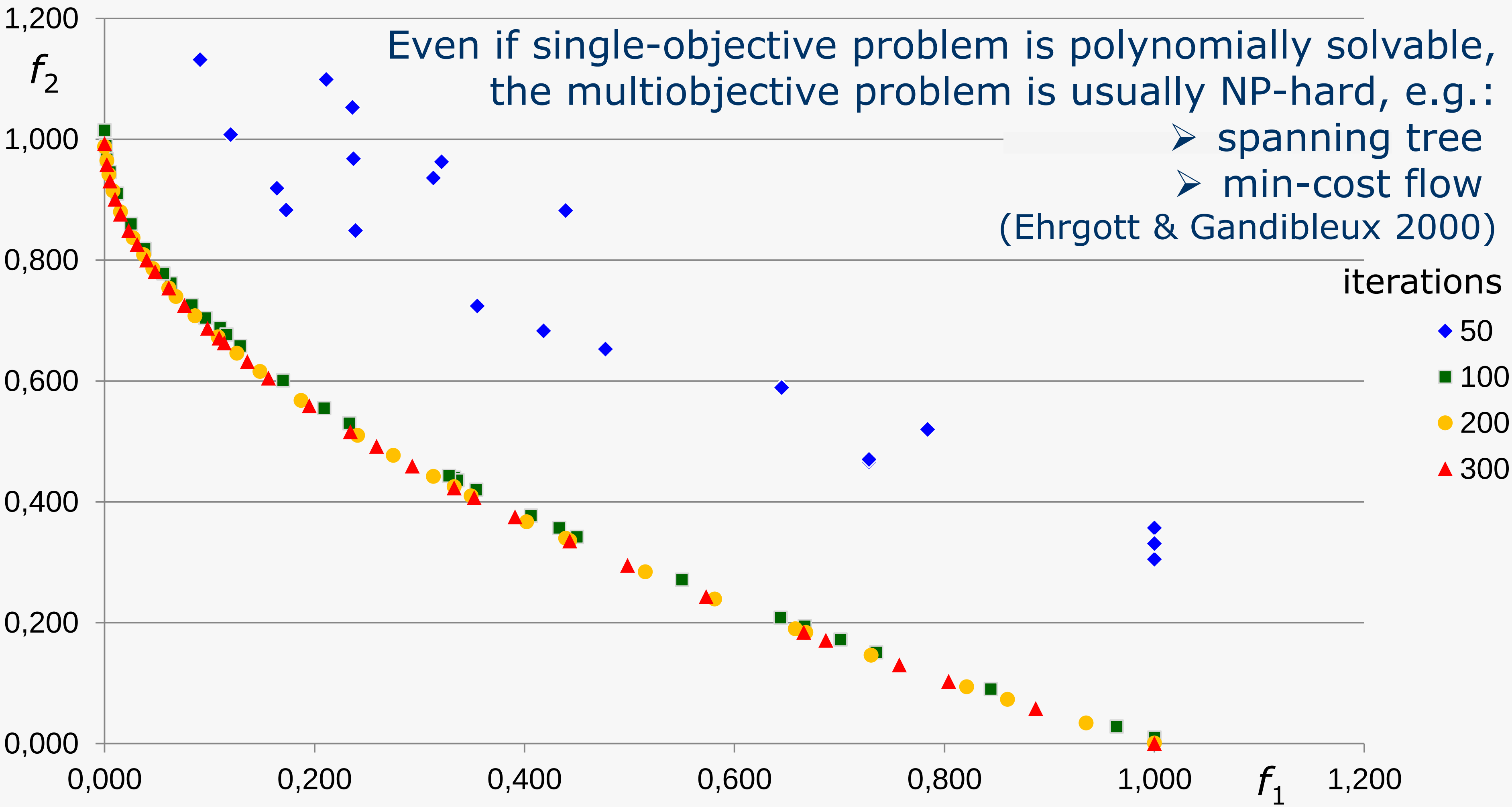
Evolutionary Multiobjective Optimization (EMO)

MOCO problems are NP-hard, #P-hard → intractable

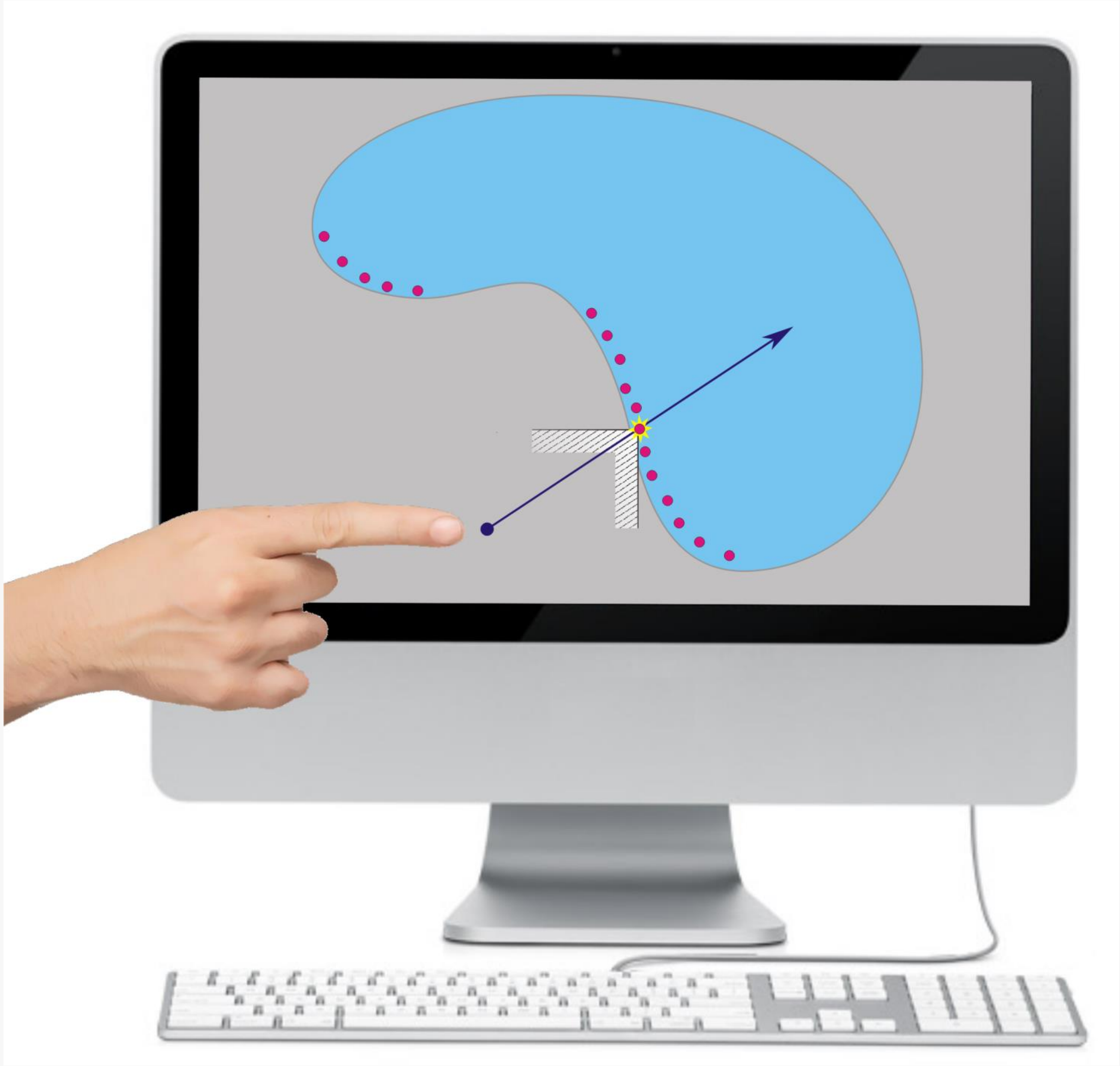
Even if single-objective problem is polynomially solvable, the multiobjective problem is usually NP-hard, e.g.:

- spanning tree
- min-cost flow

(Ehrgott & Gandibleux 2000)



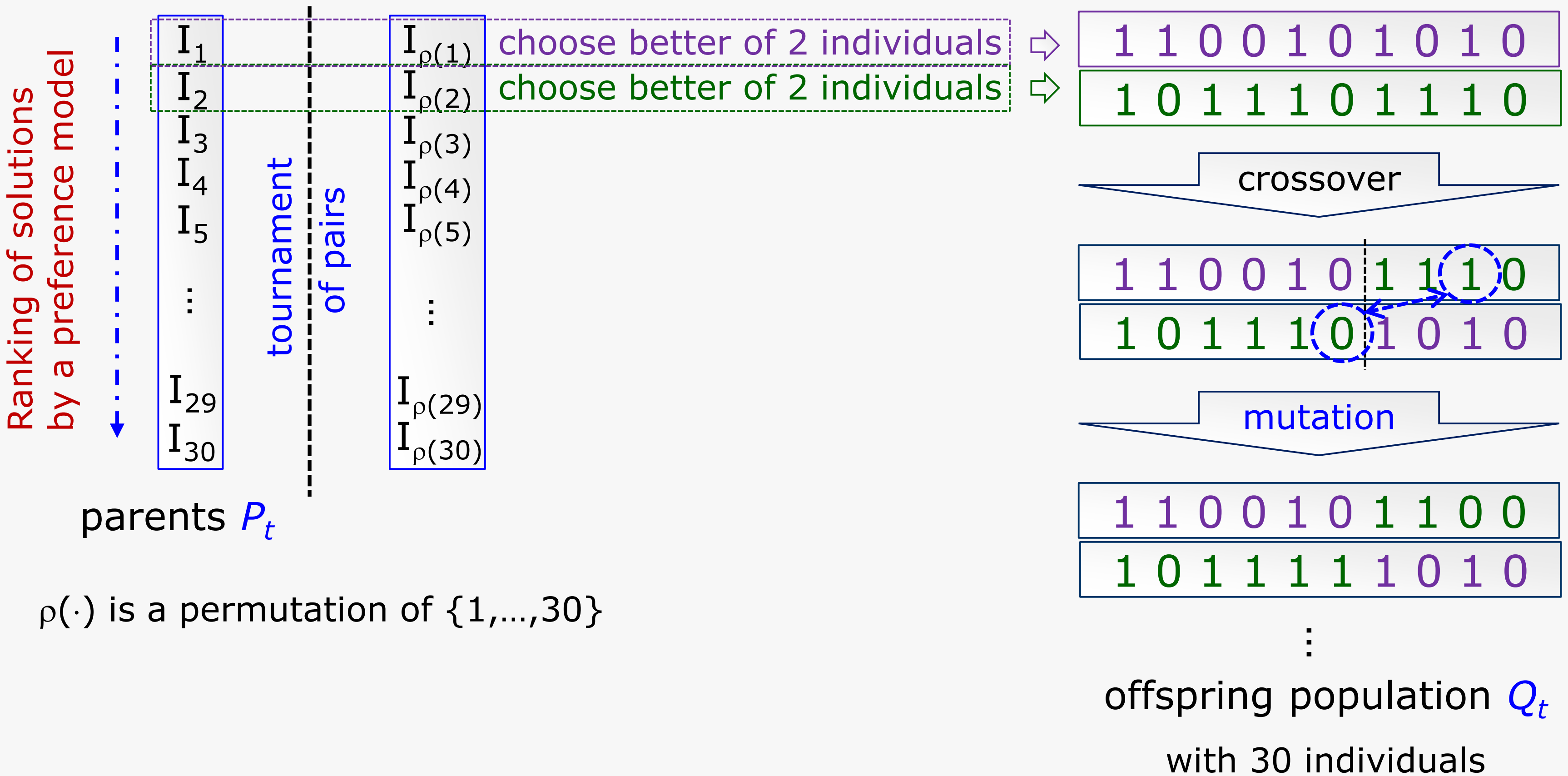
Multiobjective Optimization – „most preferred" solution



From preference model to ranking of solutions in a population

Preference pressure in the recombination procedure

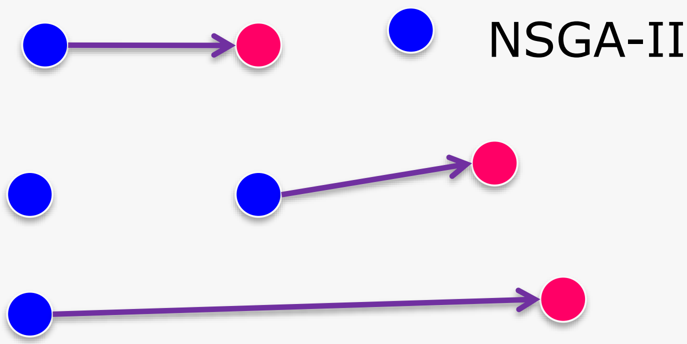
- Mating selection, crossover and mutation in generation t:



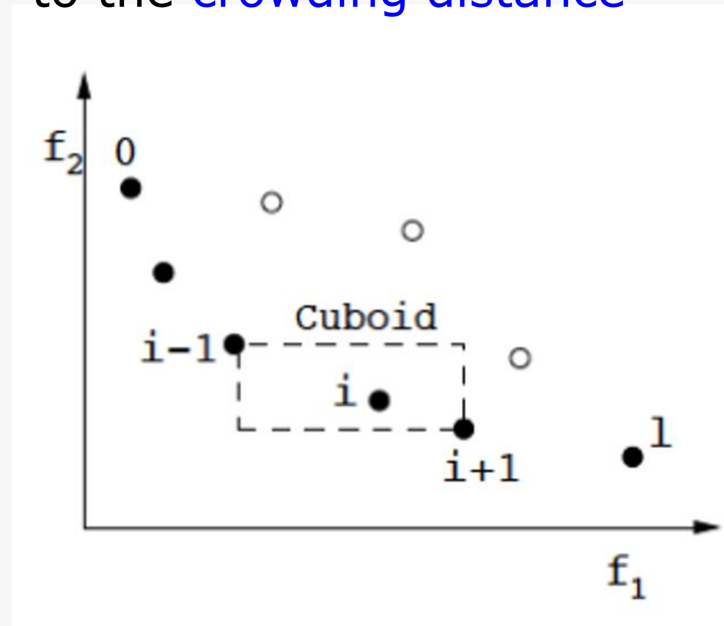
ρ(·) is a permutation of {1,…,30}

The NSGA-II framework

- NSGA-II: dominance ranking of solutions from a current population

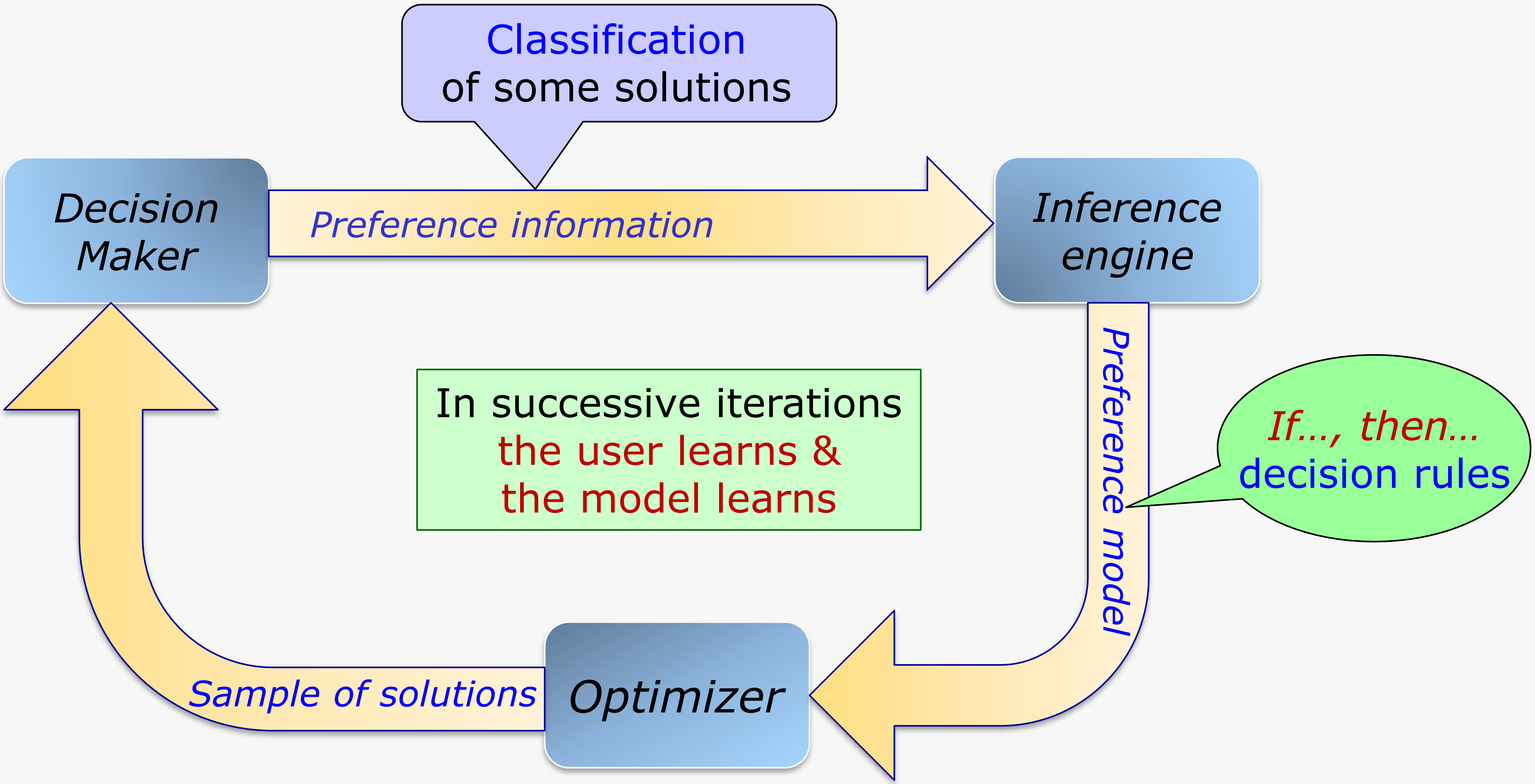


Within the same front, order the solutions with respect to the crowding distance



K. Deb, S. Agrawal, A. Pratap, T. Meyarivan: A fast and elitist multi-objective genetic algorithm: NSGA-II. *IEEE Trans. Evolutionary Computations*, 6 (2002) 182–97

XIMEA-DRSA: Interactive EMO driven by decision rules

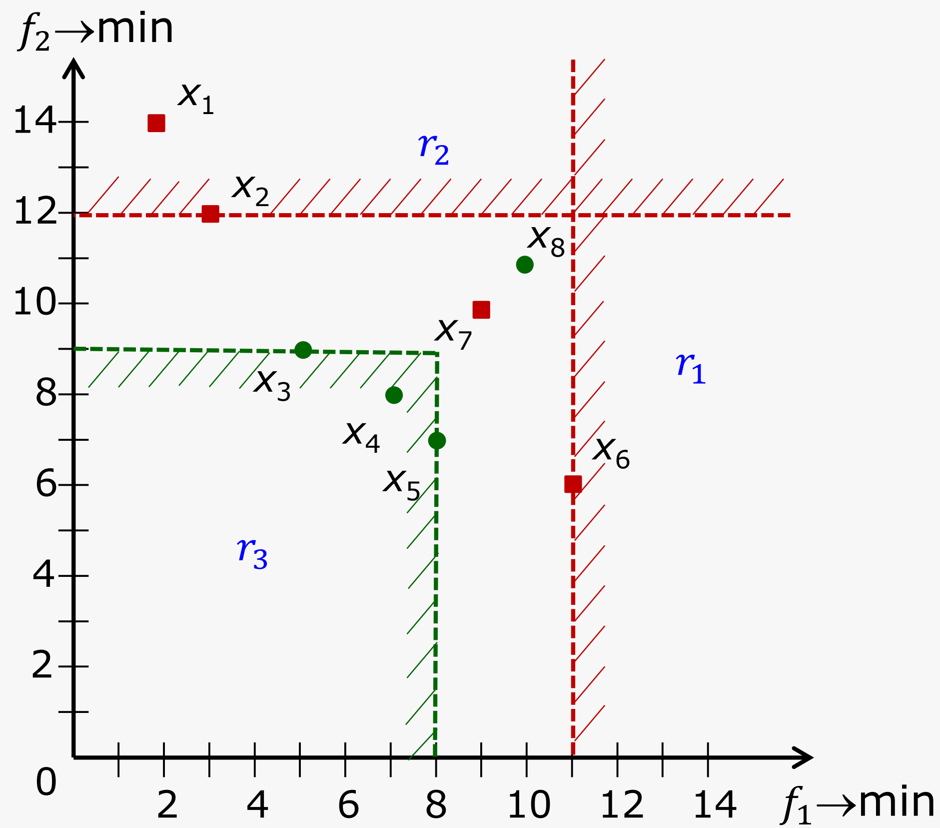


S. Corrente, S. Greco, B. Matarazzo, R. Słowiński: Explainable Interactive Evolutionary Multiobjective Optimization, *OMEGA*, 122 (2024) 102925

Example of preference information and preference model

Sample of 8 actions – induction of certain decision rules

action	f_1	f_2	DM
x_1	2	14	bad
x_2	3	12	bad
x_3	5	9	good
x_4	7	8	good
x_5	8	7	good
x_6	11	6	bad
x_7	9	10	bad
x_8	10	11	good



$D_{\leq}$ { r_1: if $f_1(x) \geq 11$, then x is *certainly* bad — supported by $\{x_6\}$

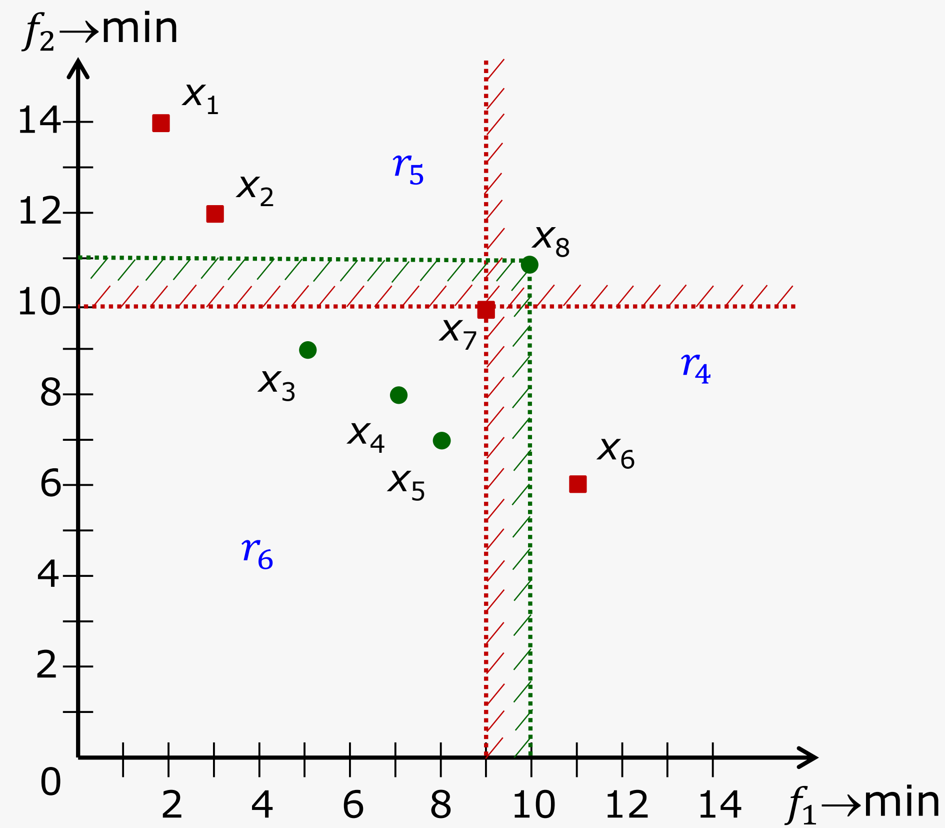
r_2: if $f_2(x) \geq 12$, then x is *certainly* bad — supported by $\{x_1, x_2\}$ }

$D_{\geq}$ { r_3: if $f_1(x) \leq 8$ & $f_2(x) \leq 9$, then x is *certainly* good — supported by $\{x_3, x_4, x_5\}$ }

Example of preference information and preference model

Sample of 8 actions – induction of possible decision rules

action	f_1	f_2	DM
x_1	2	14	bad
x_2	3	12	bad
x_3	5	9	good
x_4	7	8	good
x_5	8	7	good
x_6	11	6	bad
x_7	9	10	bad
x_8	10	11	good



$D_{\leq}$ r_4: if $f_1(x) \geq 9$, then x is *possibly* bad — supported by $\{x_6, x_7, x_8\}$

$D_{\leq}$ r_5: if $f_2(x) \geq 10$, then x is *possibly* bad — supported by $\{x_1, x_2, x_7, x_8\}$

$D_{\geq}$ r_6: if $f_1(x) \leq 10$ & $f_2(x) \leq 11$, then x is *possibly* good — supported by $\{x_3, x_4, x_5, x_7, x_8\}$

XIMEA-DRSA: Interactive EMO driven by decision rules

1. Assign solutions to ordered non-dominated fronts $NDF_1, \dots, NDF_q, \dots$
2. Inside the same non-dominated front:

   a) Calculate for each solution x the score

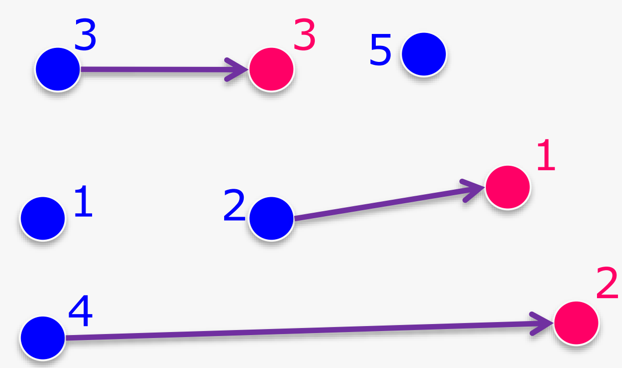
$$score(x) = \sum_{r \in D_{\geq}(x)} e^{-\gamma(t-age(r))} - \sum_{r \in D_{\leq}(x)} e^{-\gamma(t-age(r))}$$

   where $D_{\geq}(x)$ - the set of rules of type $D_{\geq}$ matching x (good rules),
   $D_{\leq}(x)$ - the set of rules of type $D_{\leq}$ matching x (bad rules)

   b) Order solutions in each NDF_q from the highest to the lowest $score(x)$

t – iteration, r – rule
$age(r)$ – the iteration in which rule r was born
$\gamma > 0$ – coefficient of the aging speed

$$NDF_q \rightarrow \begin{cases} x^1 \\ \cdots \\ x^l \end{cases} \quad \text{such that: } x^1 \cup \cdots \cup x^l = NDF_q,\ score(x^1) > \cdots > score(x^l)$$



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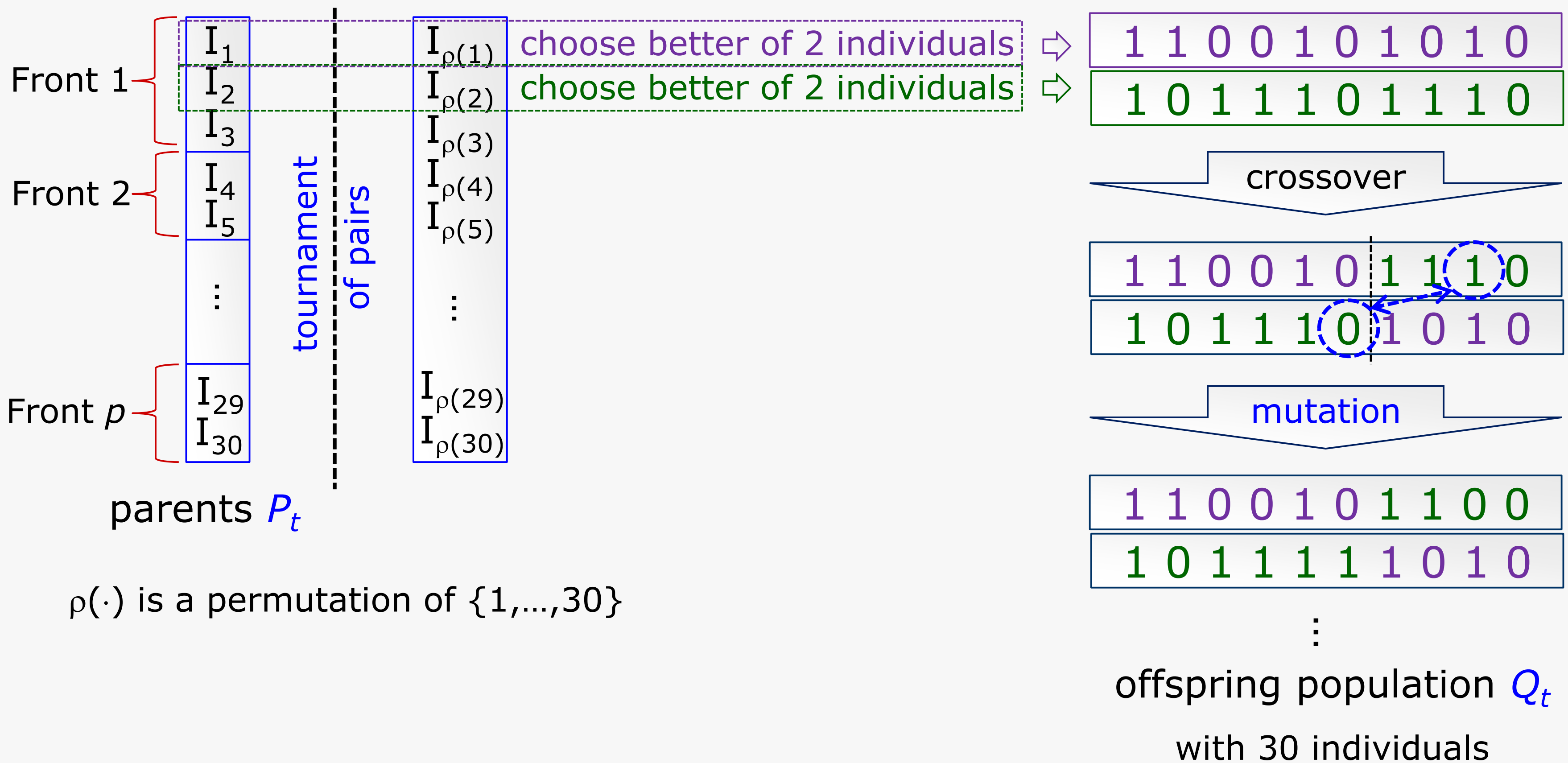
From preference model to ranking of solutions in a population

Preference pressure in the recombination procedure

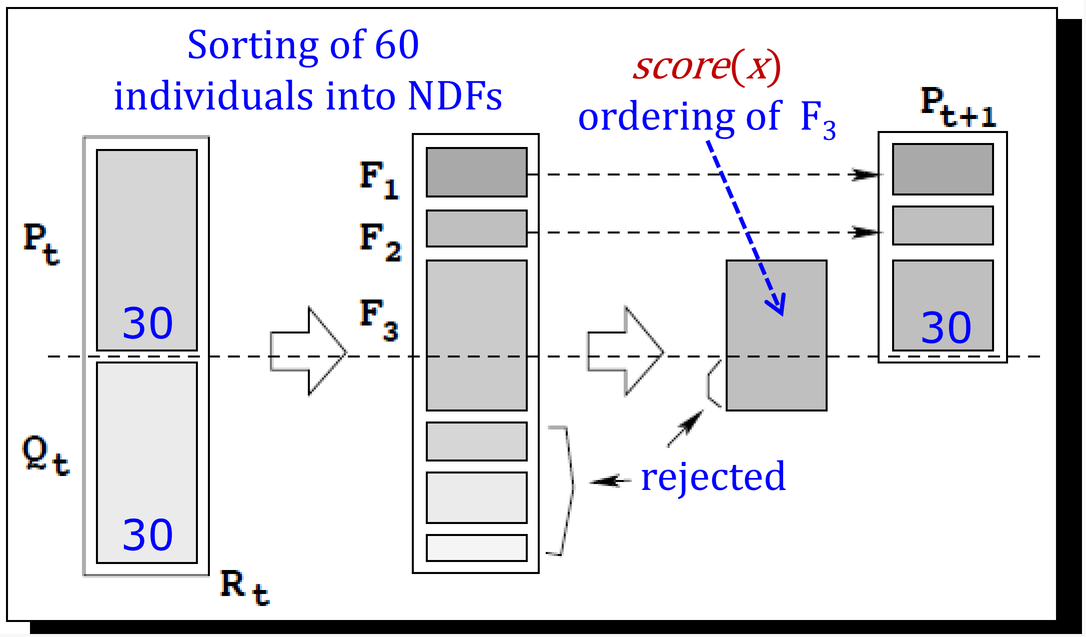
- Mating selection, crossover and mutation in generation t:



$\rho(\cdot)$ is a permutation of $\{1,\ldots,30\}$

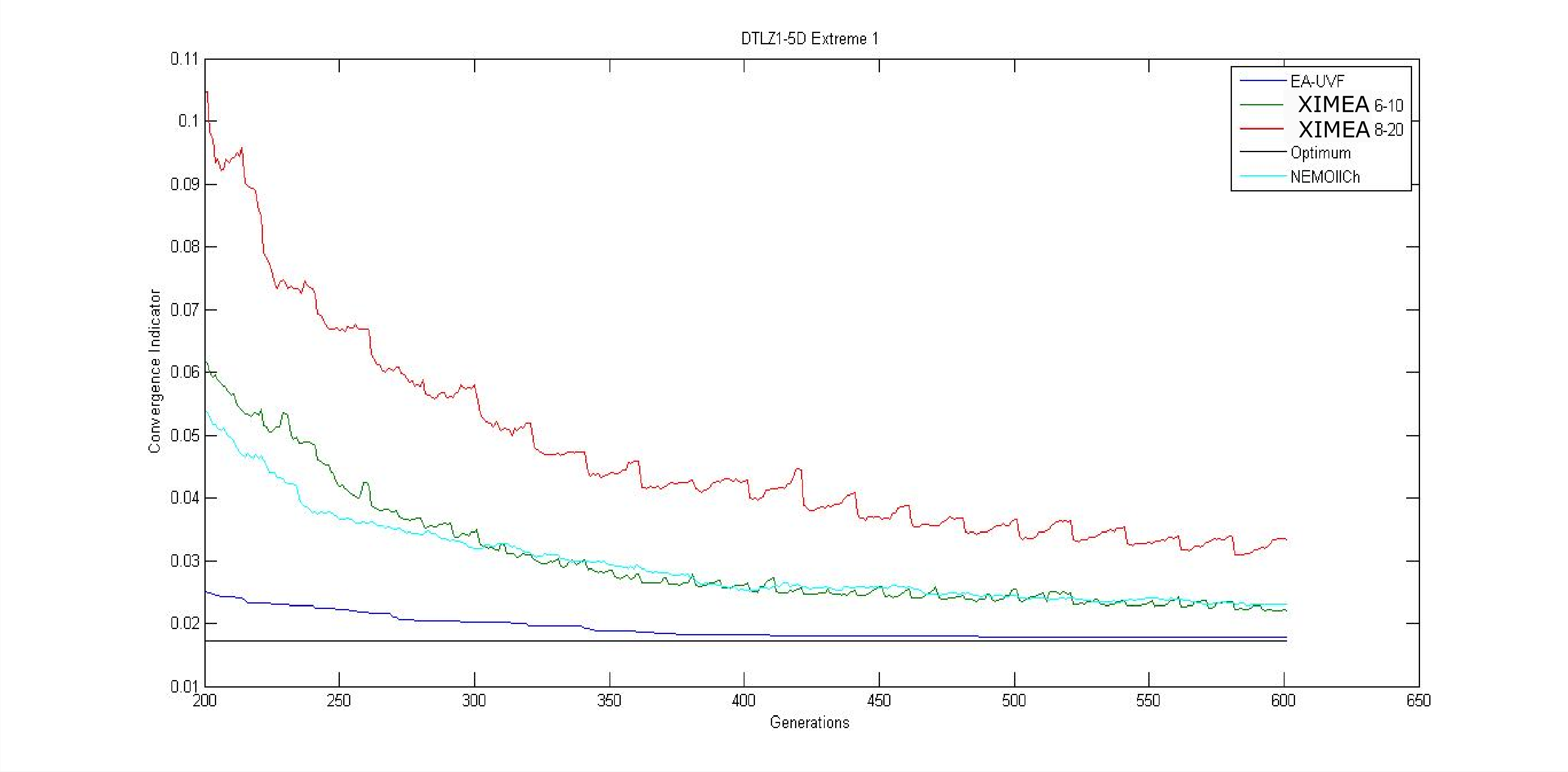
From preference model to ranking of solutions in a population

- Selection of new population P_{t+1}:

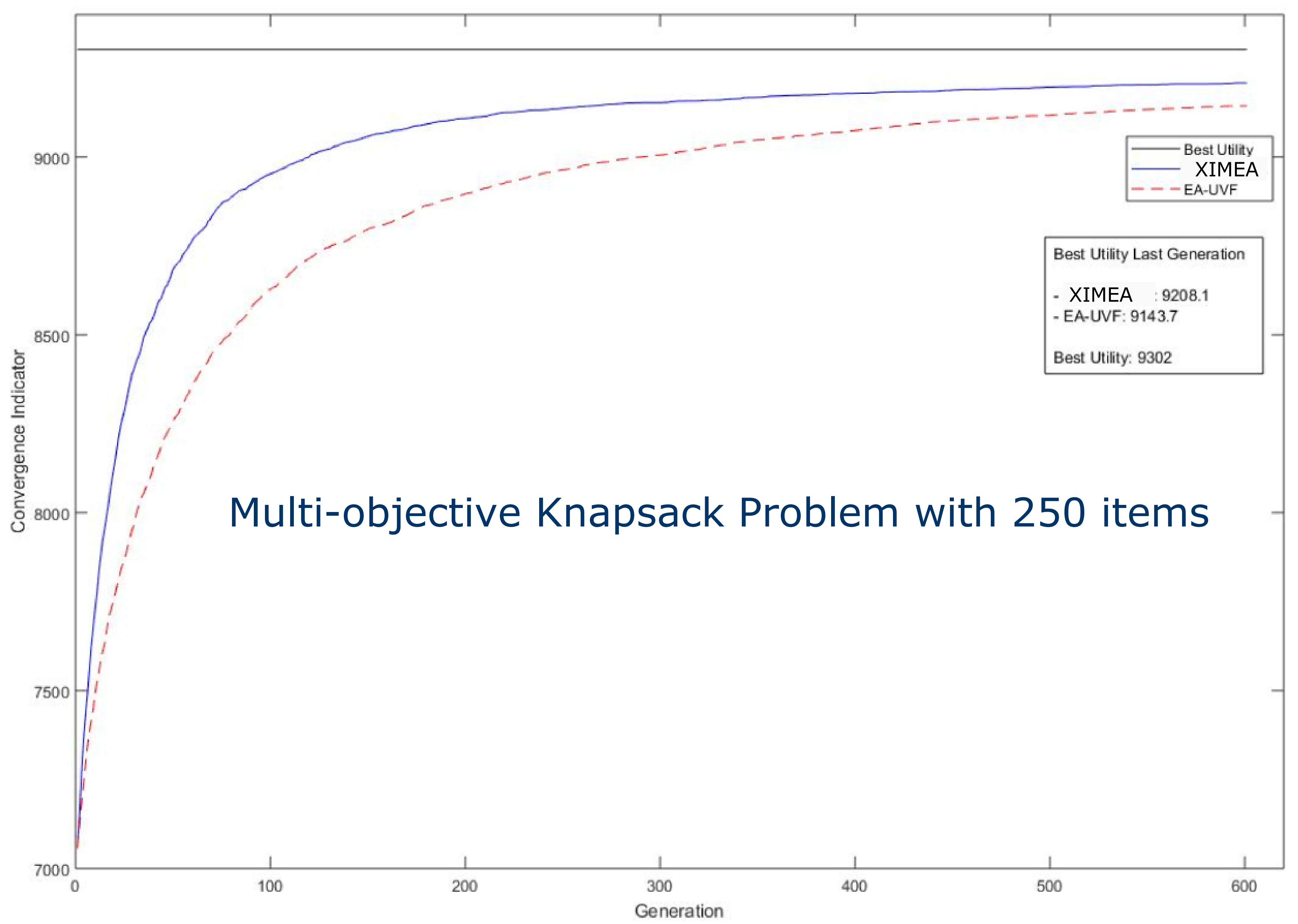


DTLZ1-5D: $U(\boldsymbol{x})=\max\{w_1 \times f_1(\boldsymbol{x}), \ldots, w_5 \times f_n(\boldsymbol{x})\} \rightarrow \min$

	w_1	w_2	w_3	w_4	w_5
DTLZ2-5D Cheb. (extreme 1)	0.1	0.15	0.2	0.25	0.3



250 items: $U(\boldsymbol{x}) = \min\{\boldsymbol{w}_1 \times \boldsymbol{z}_1(\boldsymbol{x}) - \alpha, \boldsymbol{w}_2 \times \boldsymbol{z}_2(\boldsymbol{x}) - \alpha\} \rightarrow \max$



XIMEA-DRSA: the explainability issue

- Consider the 2D knapsack problem with 100 items, $\boldsymbol{w}_2{}^1 = (1,1)$, and α=3200
- The DM is asked every 25 iterations to classify 6 current solutions into *good* or *bad* class
- From this preference information, decision rules are induced to explain the judgments of the DM
- To show how XIMEA-DRSA explains the DM judgments, let's consider iterations no.: 1, 101, and 576
- Reference solutions:

Iteration 1				Iteration 101				Iteration 576			
Sol	$f_1(\cdot)$	$f_2(\cdot)$	Classification	Sol	$f_1(\cdot)$	$f_2(\cdot)$	Classificaiton	Sol	$f_1(\cdot)$	$f_2(\cdot)$	Classification
$\mathbf{x}_1^1$	2908	3002	*good*	$\mathbf{x}_1^{101}$	3749	3863	*bad*	$\mathbf{x}_1^{576}$	3828	3827	*good*
$\mathbf{x}_2^1$	3048	2906	*good*	$\mathbf{x}_2^{101}$	3851	3786	*bad*	$\mathbf{x}_2^{576}$	3948	3758	*bad*
$\mathbf{x}_3^1$	2890	2991	*good*	$\mathbf{x}_3^{101}$	3762	3853	*bad*	$\mathbf{x}_3^{576}$	3879	3784	*bad*
$\mathbf{x}_4^1$	3042	2868	*bad*	$\mathbf{x}_4^{101}$	3816	3809	*good*	$\mathbf{x}_4^{576}$	3714	3884	*bad*
$\mathbf{x}_5^1$	2947	2803	*bad*	$\mathbf{x}_5^{101}$	3790	3837	*good*	$\mathbf{x}_5^{576}$	3804	3833	*bad*
$\mathbf{x}_6^1$	3012	2769	*bad*	$\mathbf{x}_6^{101}$	3829	3799	*good*	$\mathbf{x}_6^{576}$	3831	3818	*bad*

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XIMEA-DRSA: the explainability issue

- DRSA decision rules induced after the 1st iteration, shown to DM:
  - **rule 1_1**: $\boxed{\text{If } f_1(\mathbf{x}) \geqslant 3048, \text{ then } \mathbf{x} \text{ is } \textit{good}}$ (supported by $\mathbf{x}_2^1$),
  - **rule 2_1**: $\boxed{\text{If } f_2(\mathbf{x}) \geqslant 2906, \text{ then } \mathbf{x} \text{ is } \textit{good}}$ (supported by $\mathbf{x}_1^1$, $\mathbf{x}_2^1$ and $\mathbf{x}_3^1$),
  - **rule 3_1**: If $f_2(\mathbf{x}) \leqslant 2868$, then $\mathbf{x}$ is *bad* (supported by $\mathbf{x}_4^1$, $\mathbf{x}_5^1$ and $\mathbf{x}_6^1$).

- Decision rules are using reduced number of objectives and are not anonymous

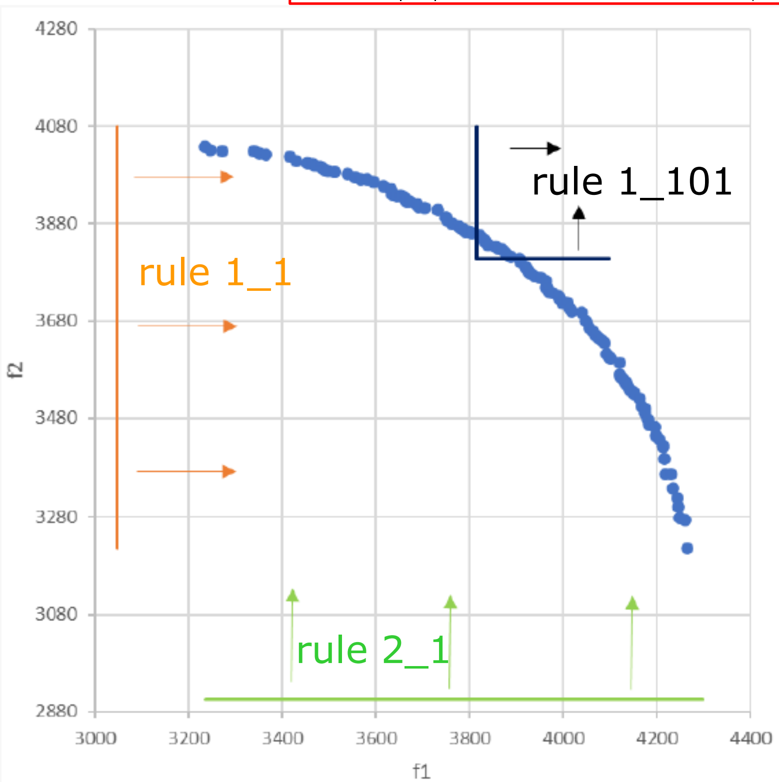
- While being transparent and intelligible, the rules used in optimization are also traceable

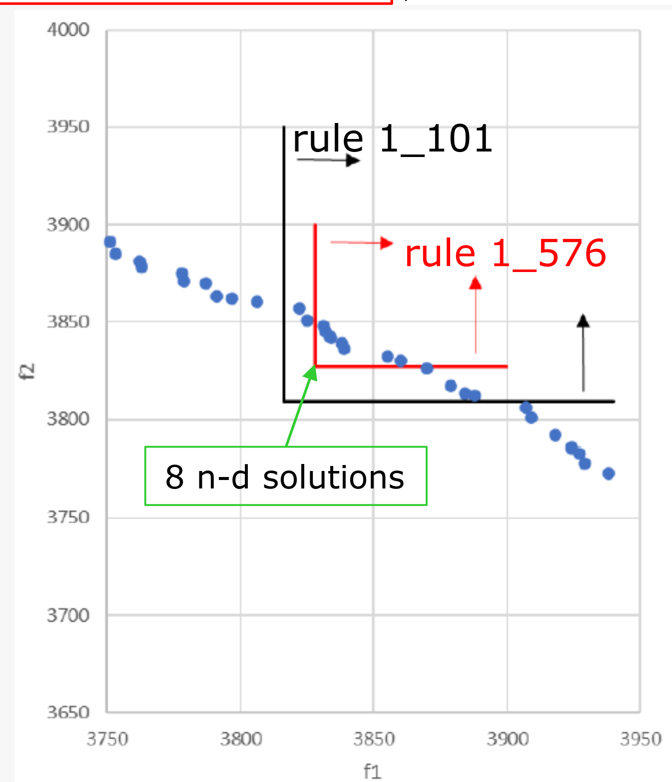
- Reflecting on the decision rules, the DM learns her preferences

- Kahneman's fast and slow thinking: rules support slow learning of preferences expressed intuitively by fast decisions

XIMEA-DRSA: the explainability issue

- *Good* decision rules induced after the 101st and 576th iteration:
  - **rule 1_101**: If $f_1(\mathbf{x}) \geqslant 3816$ and $f_2(\mathbf{x}) \geqslant 3809$, then $\mathbf{x}$ is *good* (supported by $\mathbf{x}_4^{101}$)
  - **rule 2_101**: If $f_1(\mathbf{x}) \geqslant 3790$ and $f_2(\mathbf{x}) \geqslant 3837$, then $\mathbf{x}$ is *good* (supported by $\mathbf{x}_5^{101}$)
  - **rule 3_101**: If $f_1(\mathbf{x}) \geqslant 3829$ and $f_2(\mathbf{x}) \geqslant 3799$, then $\mathbf{x}$ is *good* (supported by $\mathbf{x}_6^{101}$)
  - **rule 1_576**: If $f_1(\mathbf{x}) \geqslant 3828$ and $f_2(\mathbf{x}) \geqslant 3827$, then $\mathbf{x}$ is *good* (supported by $\mathbf{x}_1^{576}$)





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XIMEA-DRSA: the explainability issue

- Decision rules used to assign a score to the considered solutions induced from classification decisions provided up to iteration 600:

- **rule 1_600**: If $f_1(\mathbf{x}) \geqslant 3799$ and $f_2(\mathbf{x}) \geqslant 3828$, then $\mathbf{x}$ is *good* ($born(\textbf{rule 1\_600}) = 126$)
- **rule 2_600**: If $f_1(\mathbf{x}) \geqslant 3841$ and $f_2(\mathbf{x}) \geqslant 3813$, then $\mathbf{x}$ is *good* ($born(\textbf{rule 2\_600}) = 151$)
- **rule 3_600**: If $f_1(\mathbf{x}) \geqslant 3805$ and $f_2(\mathbf{x}) \geqslant 3820$, then $\mathbf{x}$ is *good* ($born(\textbf{rule 3\_600}) = 151$)
- **rule 4_600**: If $f_1(\mathbf{x}) \geqslant 3831$ and $f_2(\mathbf{x}) \geqslant 3818$, then $\mathbf{x}$ is *good* ($born(\textbf{rule 4\_600}) = 151$)
- **rule 5_600**: If $f_1(\mathbf{x}) \geqslant 3808$ and $f_2(\mathbf{x}) \geqslant 3819$, then $\mathbf{x}$ is *good* ($born(\textbf{rule 5\_600}) = 176$)
- **rule 6_600**: If $f_1(\mathbf{x}) \geqslant 3828$ and $f_2(\mathbf{x}) \geqslant 3827$, then $\mathbf{x}$ is *good* ($born(\textbf{rule 6\_600}) = 576$)

- **rule 7_600**: If $f_2(\mathbf{x}) \leqslant 3727$, then $\mathbf{x}$ is *bad* ($born(\textbf{rule 7\_600}) = 51$),
- **rule 8_600**: If $f_2(\mathbf{x}) \leqslant 3759$, then $\mathbf{x}$ is *bad* ($born(\textbf{rule 8\_600}) = 76$),
- **rule 9_600**: If $f_1(\mathbf{x}) \leqslant 3790$, then $\mathbf{x}$ is *bad* ($born(\textbf{rule 9\_600}) = 126$),
- **rule 10_600**: If $f_2(\mathbf{x}) \leqslant 3785$, then $\mathbf{x}$ is *bad* ($born(\textbf{rule 10\_600}) = 126$)
- **rule 11_600**: If $f_1(\mathbf{x}) \leqslant 3800$, then $\mathbf{x}$ is *bad* ($born(\textbf{rule 11\_600}) = 151$)
- **rule 12_600**: If $f_2(\mathbf{x}) \leqslant 3803$, then $\mathbf{x}$ is *bad* ($born(\textbf{rule 12\_600}) = 151$)
- **rule 13_600**: If $f_1(\mathbf{x}) \leqslant 3805$, then $\mathbf{x}$ is *bad* ($born(\textbf{rule 13\_600}) = 176$)
- **rule 14_600**: If $f_2(\mathbf{x}) \leqslant 3794$, then $\mathbf{x}$ is *bad* ($born(\textbf{rule 14\_600}) = 176$)
- **rule 15_600**: If $f_1(\mathbf{x}) \leqslant 3818$, then $\mathbf{x}$ is *bad* ($born(\textbf{rule 15\_600}) = 576$)
- **rule 16_600**: If $f_2(\mathbf{x}) \leqslant 3819$, then $\mathbf{x}$ is *bad* ($born(\textbf{rule 16\_600}) = 576$)

x=[3828, 3827] matches red rules & gets max score

It is also the best w.r.t. the true user's value funct.

Summary and conclusions

Summary and conclusions

- Robust Ordinal Regression is a constructive way of learning DM's preferences
- It underlines the evolution of OR and DA towards the AI paradigm of learning
- It is also a representative of the European School of Decision Aiding, because it goes along with the recommendation of its founder:

Bernard Roy (1934-2017): „MCDA must be based on models that are co-constructed through interaction with the decision maker.
The co-constructed model must be a tool for looking deeper into the subject, exploring, interpreting, debating and even arguing." (2010)

Thank you for your attention

Acknowledgment to co-operators:

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